SPECKLE

Nicholas George

Speckle has been a great nuisance in the field of holography. In optical readers and other optical processors, too, we have learned to provide a "smooth" input format or else suffer the disaster of speckle. These limitations have been understood for a number of years. Still there is no simple, direct cure for speckle, and logically, no hope for one. Why then a Topical Meeting on Speckle at Asilomar? To be sure, it is in part to review these basic suppression techniques, even though we understand them fairly well. More importantly, the meeting is planned to discuss this basic noise phenomenon in optics with today's perspective. Recent formulations of the theory provide generality of result and ease of understanding. The primary goal,

The author is with the Department of Applied Physics, California Institute of Technology, Pasadena, California 91125.

however, is to bring together scientists who are now working on a variety of newer optical systems—systems in which speckle is a useful entity, or where speckle-notions have guided the system concept, or where the use of partially coherent light is a convenient way around speckle.

Among these new disciplines is speckle interferometry. In astronomy, Labeyrie has successfully reduced the blur resulting from atmospheric turbulence. Interesting work is currently under way on image retrieval, real-time processing, large arrays of optical elements, and adaptive systems.

Speckle interferometry is also leading to methods for measuring small displacements of diffuse objects. Related techniques for measuring translation, rotation, and tiny vibration are being studied at numerous laboratories. Tunable-laser sources are being applied to the measurement of surface roughness, to the remote sensing of object shape as well as roughness, and to basic studies of resolution. In microwaves and acoustics, analogous speckle problems and system innovations are being explored at various laboratories.

The program of the Asilomar Topical Meeting on Speckle Phenomena in Optics, Microwaves, and Acoustics begins on page 31.

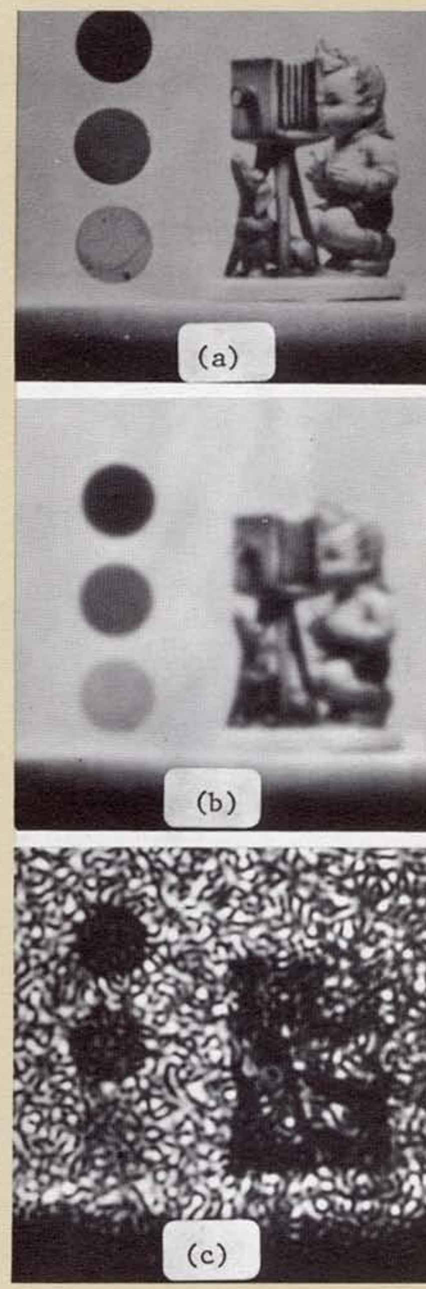


Figure 1. The origin of speckle. A reference input transparency, backed by an opal glass diffuser, is copied in a simple imaging system using various illuminations and pupil diameters. (a) shows the test object at high resolution, *f*/8, using white light. The disks have neutral densities of 0.3, 0.6, and 1.0, starting from the bottom. (b) shows the resolution in white light at *f*/190 and can be compared directly with (c) for the speckle-limited case where a single-moded argon laser is used, again at *f*/190.

WHAT IS SPECKLE?

The word *speckle* is now used to describe not only the sparkling, granular, and highly contrasted appearance of visible laser light reflected from a wall or other diffuse surface; but its usage has been generalized to include analogous spatial interference effects that occur with scattering from diffuse objects for all types of wave-motion phenomena.

Speckle constitutes a basic noise phenomenon limiting the working resolution of coherent systems in acoustics, microwaves, and infrared and visible optics; and undoubtedly its manifestations in electron-particle waves and ultraviolet will be recognized shortly.

Generally, any wave with a narrow angular spectrum that is scattered from a diffuse object will exhibit speckle if:

1. the spectral width is below a certain value and
2. the object has textural roughness finer than, but on the order of, the size of a resolution cell or, more grossly, on the order of a wavelength.

The origin of speckle is shown in Figure 1. Illustrating the test pattern used, Figure 1(a) shows a copy at $f/8$ of a positive transparency of a statuette that also has three equal-diameter discs with neutral density values, reading from the bottom to the top, of 0.3, 0.6, and 1.0. An opal glass diffuser is placed between the light source and the input transparency in order to create the speckle. Also, to make it large, the following speckle photos are taken at $f/190$, unless otherwise noted. Hence, the proper comparison with white light, unspeckled, is the rather blurred reproduction in Figure 1(b).

Figure 1(c) shows the badly speckled image resulting from processing the diffuser-transparency combination in monochromatic light. This combination is a good simulation for the speckle problems that are inherent in the use of coherent processors for imagery on ordinary, rough paper or in recording holographically a diffusely reflecting object. It is particularly interesting to note the poor resolution for low-contrast portions of the subject.

Viewing the speckle pattern as random noise that obscures the desired image, one would expect an improved result if separate looks are averaged. These independent looks can be obtained in a variety of ways.[1-5] Figure 2(a) shows the effect of using an imaging system with a pupil that is much larger than is required by the object's detail.[1] Figure 2(b) shows the excellent suppression that results when a slowly moving diffuser is used in cascade with a motionless one.[4] Figure 3 shows the result of superimposing speckle patterns taken at different wavelengths.[5]

THE PROBLEM OF THE RANDOM WALK IS CENTRAL TO UNDERSTANDING SPECKLE PHENOMENA

"A man starts from a point 0 and walks L yards in a straight line; he

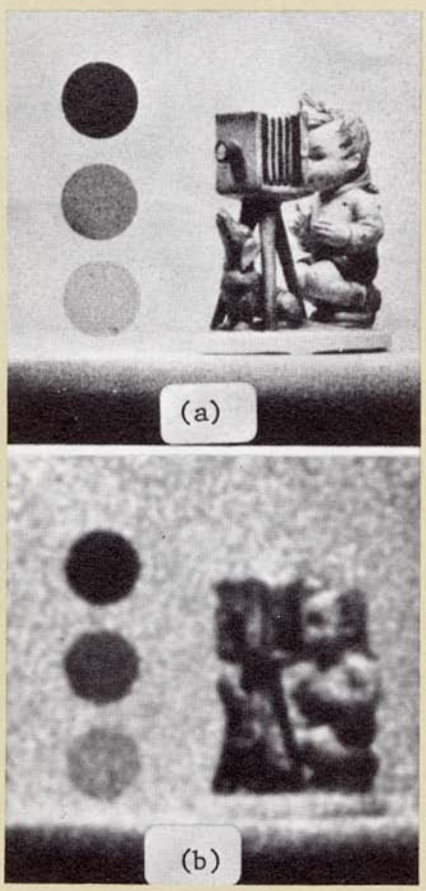


Figure 2. Two basic methods for providing redundancy, hence smoothing of speckle, are shown using the same input transparency, diffuser, and one-tone laser source as in Figure 1 (c). Photograph (a) is prepared by using a large aperture ($f/8$) relative to the spatial content of the object; and (b) uses cascaded diffusers with one in motion, again at $f/190$.

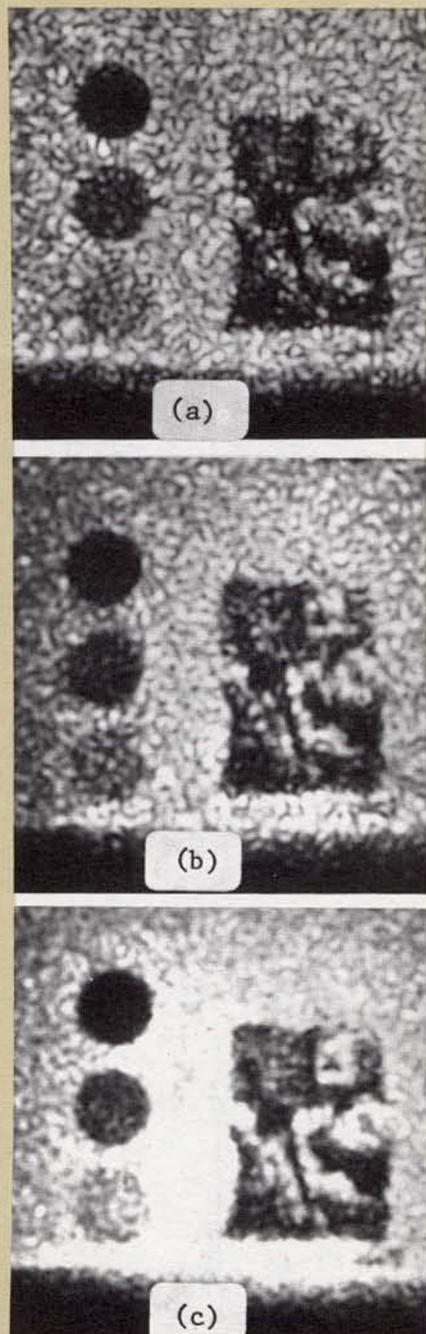


Figure 3. Smoothing of speckle is obtained with independent looks provided by multiple wavelengths from a dye laser source; (a) 2, (b) 4, and (c) 16 wavelengths are used. An interval of 25 Å is used, although for the opal glass diffuser 15 Å is adequate to decorrelate the speckle. These photos should be compared with the badly speckled case in Figure 1 (c) and the white-light case at $f/190$ in Figure 1 (b). All were taken at $f/190$ with plane-polarized illumination and a parallel analyzer in the recording.

then turns through any angle whatever and walks another L yards in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $r + dr$ from his starting point."[6]

When a monochromatic electromagnetic wave is reflected from a rough surface, which, not incidentally, has some imprinted text, the scattered electric field at an arbitrary observation point is analogous to the total distance in the problem of a random walk. The reason for this is that the total field is the superposition of components scattered by the infinitesimal surface contributors; these are at relative phases that fluctuate rapidly if the local surface heights vary by dimensions on the order of a wavelength. The phase angles of the individual terms correspond, of course, to the different directions in the random walk.

However, the speckle problem is inherently more complex than the random walk, as we can see by considering a neighboring observation point. In calculating the total field, the phase delay for each infinitesimal of the scattering surface depends upon its radian-distance to the point of observation. Thus, if the neighboring point is displaced sufficiently, one would expect the total field obtained by this summation to differ appreciably from its value at the initial point. This is the idea of spatial decorrelation of speckle patterns, or more simply of speckle size. A sound quantitative understanding of the details would result from an application of Fresnel-zone ideas, but this analysis is not pertinent herein.

For transmission through a rough, phase-type diffuser, approximate formulas for the average speckle size on-axis in the Fresnel zone are given in Figure 4. This jelly-bean appearance of the speckles changes in the transition to the far-zone region, i.e., the the longitudinal variation, $d_{||}$, disappears as $R \to \infty$. However, the transverse dimension continues to subtend the fixed angle λ/D, where λ is the wavelength and D is the aperture size. Finally, since this radian-distance to the observation point also depends inversely on the wavelength, we understand immediately that speckle patterns should exhibit considerable sensitivity to wavelength changes.

COMMUNICATION THEORY, CORRELATION FUNCTIONS, FOURIER TRANSFORMS, AND THE LIKE

A deeper insight into speckle was obtained recently by several investigators working separately and following Burckhardt's analysis of speckle statistics based upon the formalism, and hence utilizing well known theorems, of communication theory.[7]

A brief analytical statement about speckle serves to illustrate this idea most directly as well as to introduce some of the functions that occur. Two different speckle problems are reviewed. In one, the speckle results from a pure phase diffuser illuminated in transmission and imaged by a lens of finite diameter D. The speckle in the image plane is described. In the second, the speckle results from scattering or reflection of laser light by a rough surface.

First, consider the image plane case. The input diffuser is modeled by the transmission function exp $[-i\eta\, \rho(x)]$ in which $\rho(x)$ is the random process of surface heights at the transverse position x and $\eta = 2\pi\nu\Delta n/c$ is the generalized temporal-frequency variable, where ν is the frequency of the light, Δn is the index-of-refraction difference between the diffuser and the surrounding, and $c = 3 \times 10^8\ m/s$. The output electric field $E(x,\eta)$ for imaging by a lens system with impulse response $z(x;\eta)$ is given by the following convolution:

$$E(x,\eta) = \int_{-\infty}^{\infty} z(x-x_0;\eta) \exp\left[-i\eta\, \rho\,(x_0)\right] dx_0 \qquad (1)$$

Both space and wavelength dependences of the speckle can be studied if one computes the following correlation functions:

$$R_e = \langle E(x_1,\eta_1)E^*(x_2,\eta_2)\rangle \qquad (2)$$

and

$$R_u = \langle E(x_1,\eta_1)E^*(x_1,\eta_1)\, E(x_2,\eta_2)E^*(x_2,\eta_2)\rangle \ , \qquad (3)$$

where the brackets $< \ldots >$ denote an ensemble average over the random process in texture, $\rho(x)$.

Substituting Equation (1) into Equation (2) and interchanging the order of integration and expectation lead to the form below, which contains the characteristic function F. Thus, R_e is given by

$$R_e = \iint_{-\infty}^{\infty} z^*(x_2-x'')z(x_1-x')\, F(\eta_1,-\eta_2)\, dx'dx'', \qquad (4)$$

where an input plane of infinite extent is implicitly assumed.

Following the notation used in the theory of probability, we have defined $F(\eta_1,\eta_2)$ in Equation (4) as the following expectation:

$$F(\eta_1,\eta_2) = \langle \exp\left[-i\,\eta_1 \rho(x') -i\eta_2\, \rho(x'')\right] \rangle \ , \qquad (5)$$

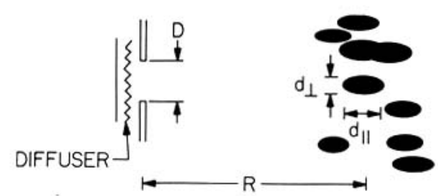
which is equivalent to the Fourier transformation of the density $f(\rho_1,\rho_2)$:

$$F(\eta_1,\eta_2) = \iint_{-\infty}^{\infty} f\left[\rho_1, \rho_2\right] \exp[-i(\eta_1\, \rho_1 + \eta_2\, \rho_2)]\, d\,\rho_1 d\,\rho_2 \ , \qquad (6)$$

where $\rho_1 = \rho(x')$, $\rho_2 = \rho(x'')$, and η_1,η_2 are the frequencies introduced by the transform operation. The roughness heights of the input diffuser are described by the joint density function $f(\rho_1, \rho_2)$.

In the literature one finds Equation (4) described as follows: "The spatial and spectral autocorrelation of the speckle field in the image plane of a linear space-invariant system is given by convolving the autocorrelation of the system's impulse response with the characteristic function for the random process of diffuser heights."[8]

I have dwelt on this topic at length in order to explain the typical occurence of the characteristic function in speckle problems. The use of the correlation function provides a precise meaning for the average speckle size, refining the discussion of Figure 4. Moreover, it provides a deeper understanding of the entire space- and wavelength-dependences of



$$d_\perp = \frac{\lambda R}{D}$$

$$d_{||} = \frac{4\lambda R^2}{D^2}$$

$$\frac{\Delta\lambda}{\lambda} > \frac{1}{2\pi\Delta n\,(h_0/\lambda)}$$

Figure 4. Approximate characterization of speckle, on axis, in the Fresnel-zone region for monochromatic illumination incident on a rough diffuser. Transverse and longitudinal speckle sizes can be calculated from the above formulas for $d_\perp$ and $d_{||}$, respectively. The wavelength interval required for decorrelation, denoted by $\Delta\lambda$, is given in terms of h_0, the r-m-s roughness of the diffuser, and the index interface Δn between the diffuser and its surroundings.

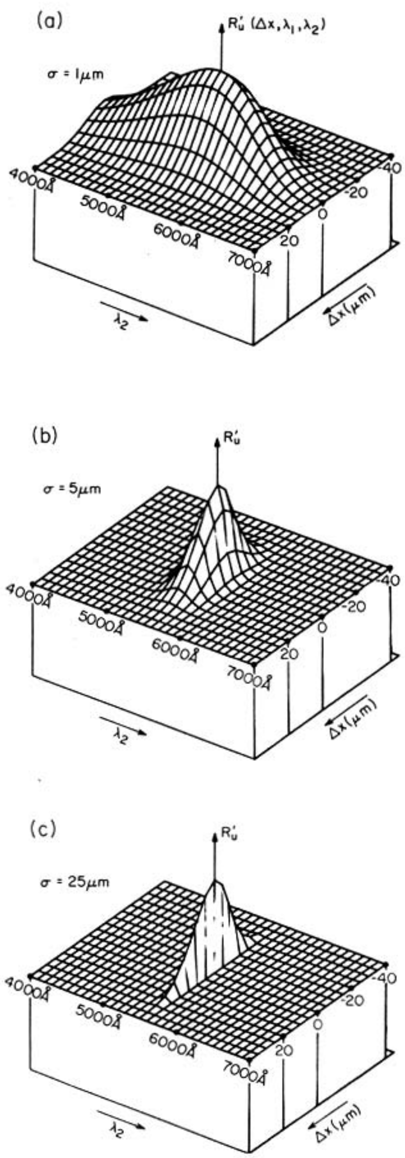


Figure 5. The correlation function for intensity in the image plane of a phase-type diffuser is plotted versus distance offset and wavelength. With a fixed aperture and correlation length on the diffuser, the r-m-s roughness σ is varied: (a) 1 μm, (b) 5 μm, (c) 25 μm. The wavelength decorrelation is quite sensitive to the diffuser's texture, but the speckle size is not.

speckle. For the correlation function $R_u(x_2-x_1;\lambda_1,\lambda_2)$, plotted normalized in Figure 5, its spatial dependence is largely controlled by the diameter of the imaging lens, whereas the wavelength dependence is dominated by the characteristic function of the diffuser heights.

SCATTERING FROM ROUGH OBJECTS, PARTICULARLY WHEN THEY HAVE AN UNDERLYING SHAPE, LEADS TO A NEW CLASS OF SPECKLE PROBLEMS

Now, for the second problem, consider a rough object of height $h(x) = h_0(x) + \rho(x)$, where h_0 is a general shape function and $\rho(x)$ is a random process of zero mean describing the surface texture, as in Figure 6. The y dependence is ignored for simplicity. For a monochromatic (laser) beam incident at a polar angle θ_0, one can show that the electric field scattered into the far-zone is given by an integral of the form

$$E(x_1,z_1;k) = e^{-ikr}\int_S \exp\,[ikx' \cdot (\sin\theta_0 + \sin\theta_1) + ikh(x') \cdot (\cos\theta_0 + \cos\theta_1)]dx' , \qquad (7)$$

where integration is over the surface S; and the wavenumber k and the wavelength λ are related by $k=2\pi/\lambda$. The observation point is at (x_1, o, z_1) in a cartesian system with $\sin\theta_1 = x_1/r$ and $r^2 = x_1^2 + z_1^2$. The correlation functions have a wavelength dependence functionally related to F; and their spectral intensities (transforming with respect to the wavenumber k) depend on the density function of heights $f(\rho_1, \rho_2)$. This

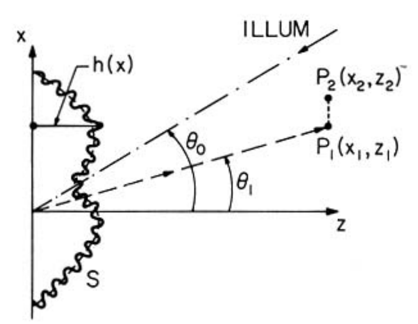


Figure 6. An interesting new class of speckle problems arises when the scattering is from an extended, rough object that has a composite height $h(x) = h_0(x) + \rho(x)$, where h_0 is the general surface profile and ρ is the texture. Illumination incident at an angle θ_0 is scattered to observation points P_1 and P_2 in the far zone.

is the basis for much current research in the remote measurement of the detailed height distribution of diffuse surfaces.

Motion of the diffuser both during and between speckle measurements is used in different optical systems. Measurement of in-plane displacement, object rotation rate, and object translation are a few examples. Although a comprehensive solution of this problem has not yet appeared, some useful conclusions can be drawn about the following questions. If the diffuse object is displaced in-plane by an amount Δx, how does the speckle pattern change? What is its final form?

This problem is treated formally by computing the cross-correlation function of the electric field or intensity with a state I and state II position of the given member of the diffuser ensemble as the expectation is calculated. But, reasoning directly, one can make the following observations. At first, the speckle pattern simply translates by the amount Δx. Thus the intensity pattern remains highly correlated, spatially, and this would show in the function R_u as a peak at $\Delta x \neq 0$. Thereafter, the pattern decorrelates spatially, but only if it is translated a rather large amount, say by $L/2$, where the dimension of the illuminated portion is L.

The reason for this is that the observed intensity is made up of a large group of individual contributors, say N. Of course, its expected value is small, since the random phasors add only to the total intensity, which would be given by $\sqrt{N}$ in-phase contributors. Still, when a tiny fraction of the N components is replaced by others chosen at random, the probability of their greatly changing the observed intensity is quite small. Thus, so-called edge effects are small.

In the rotation of compound shapes, the speckle pattern also rotates. Typically, several hundred speckles will rotate past a fixed observation point before the pattern,

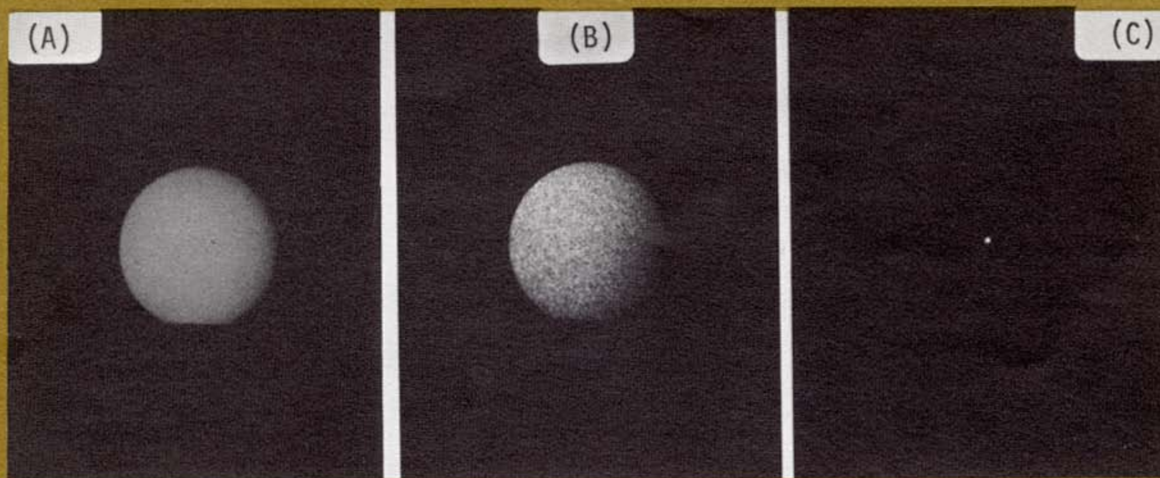


Figure 7. The possibility of remotely sensing an object's texture ρ and shape profile h_0 is described in the text. These photographs show the light reflected back from three spheres of identical diameter, 25.4 mm, as follows: (a) roughened and illuminated with white light; and then in collimated laser light are: (b) the roughened sphere, and (c) a relatively smooth sphere.

viewed globally, decorrelates. In this case, there are two factors that lead to decorrelation. The first is the removal of a fraction of the N scatters and their replacement by an uncorrelated equal number, as for the in-plane displacement. The second is that the random samples are assigned to a different phase-bin because of the change in the radian-distance between the observer and the scattering centers as the object rotates.

So in speckle metrology it is feasible to measure the rotation rate of diffuse spheres as well as the in-plane displacement or velocity of a rough sheet by counting the number of speckles that cross a fixed observer. Additionally, it is feasible to sense object dimension by studying the decorrelation angle between speckle patterns monitored at two observation points as the object rotates.

Another current topic is the remote sensing of an object's profile, i.e., $h_0(x)$, using frequency-modulation methods. For the flat surface at normal incidence, as well as for a diffuser in transmission, the texture function $\rho(x)$ dominates the temporal-frequency behavior of R_e and R_u; but once the diffuse surface is curved, then the shape function h_0 (x) becomes the dominant factor. It is readily shown that this dominance by $h_0(x)$ is particularly strong at small values of k_1-k_2, the offset frequency.

A visual comparison of the back-scattering from ball-bearings of 25-mm diameter is shown in Figure 7. When the ball is rough, as in Figures 7(a) and 7(b), there is strong return from all illuminated regions. For the smooth ball, as in Figure 7(c), the cw or steady-state return is limited to the specular zone. In the former case, it is readily understood how, by frequency modulating the laser, one can expect to deduce remotely the shape features of an object. This remote sensing is an excellent application for the narrow-band, tunable dye laser.

MORE ABOUT SPECKLE

In this article the underlying ideas about speckle have been presented in terms of first principles, rather than as a chronological review. For those who would like to see more, a movie showing space and wavelength sensitivity and object motion effects has been prepared by the author and A. C. Livanos. It is available for short-term loan upon written request.

Finally, our aim also has been to provide some insight into newer topics of speckle research. For a comprehensive listing of such topics, consult the schedule for the speckle meeting at Asilomar; it is included in this issue of *Optics News*.

REFERENCES

1. E. Leith and J. Upatnieks, J. Opt. Soc. Am. **54**, 1295 (1964).
2. J. Upatnieks, Appl. Opt. **6**, 1965 (1967).
3. H.J. Gerritsen, W.J. Hannan, and E.G. Ramberg, Appl. Opt. **7**, 2301 (1968).
4. S. Lowenthal and D. Joyeux, J. Opt. Soc. Am. **61**, 847 (1971).
5. N. George and A. Jain, Opt. Commun. **6**, 253 (1972).
6. Quotation from Pearson by J. W. Strutt, third Baron Rayleigh, *Scientific Papers* (Cambridge University Press, 1920), Vol. VI, Art. 441, p. 604.
7. C.B. Burckhardt, Bell Syst. Tech. J. **49**, 309 (1970).
8. N. George and A. Jain, Appl. Phys. **4**, 201 (1974).