Phase-Conjugate Optics

JOHN AUYEUNG and AMNON YARIV

A new research area in coherent optics has emerged and has been receiving increasing attention from many scientists as its important applications are recognized. Phase-conjugate optics is the name that seems to have been accepted to describe this new field. The main feature of phase-conjugate optics is the generation of an electromagnetic wave with a phase distribution that is, at each point in space, the exact opposite of that of an arbitrary incoming monochromatic wave. The wavefront, after being generated, proceeds to propagate in the opposite direction, retracing the original path of the incoming wave. Thus the phase-reversal or -conjugation process results in what is frequently called a time-reversed replica of the incident wave. If we consider, as an example, an incoming spherical wavefront diverging from a point with a radius of curvature R, its conjugated replica will be an outgoing spherical wavefront converging toward the same point with a radius of curvature $-R$. Phase-conjugation techniques have been used in the past for imaging through phase-distorting media; well-known examples can be found in holography[1] and adaptive optical systems.[2] The new and attractive feature that differentiates phase-conjugate optics from the previous techniques is the use of nonlinear optical mixing to generate in real time, without the need for intermediate electronics and with amplification if desired, a time-reversed replica of an incident wave.

The first experiments in phase-conjugate optics were performed in 1970 in both the USSR[3] and The Netherlands.[4] The theoretical understanding of the effect was, however, only qualitative, and the work seems to have slowed down or shifted to more-conventional holographic directions.

Using stimulated Brillouin scattering, Zeldovich *et al.*[5] demonstrated phase conjugation in the backward-scattered beam. Yariv independently proposed[6,7] the use of three-wave mixing in crystals for real-time holography and to compensate for the distortion that arises because of the model dispersion in transmitting pictorial information through a single multimode optical fiber. Phase conjugation using this technique was subsequently demonstrated by Avizonis *et al.*[8] More recently, Hellwarth[9] showed that four-wave mixing[3,4] eliminated the phase-matching restriction inherent in the three-wave process. Yariv and Pepper[10] applied the formalism of nonlinear optics to show that the four-wave process is capable of amplifying an incident wave as well as rendering its complex conjugate version, and, in the limit of sufficient pumping, of mirrorless oscillation.

The authors are with the California Institute of Technology, Pasadena, California 91125.

THEORY

We shall discuss the basic approaches proposed to date for phase conjugation. In the case of three-wave mixing, two input waves: E_1, at a frequency ω and a wavevector $\mathbf{k}_1(\omega)$, and an intense pump wave, E_2, at a frequency 2ω and a wavevector $\mathbf{k}_2(2\omega)$, are incident simultaneously on the crystal. A nonlinear polarization proportional to $E_1^*E_2$ is formed in the medium in addition to the linear polarization. This second-order nonlinear polarization, occurring only in crystals lacking inversion symmetry, acts as a source that radiates a third wave, E_3, at the difference frequency $2\omega - \omega = \omega$ and with a spatial-amplitude distribution proportional to $E_1^*E_2$. As an example,

$$E_1 = \tfrac{1}{2}A_1 \times \exp\{i[\omega t - \mathbf{k}_1(\omega)\cdot r]\} + \text{c.c.}, \tag{1}$$

$$E_2 = \tfrac{1}{2}A_2 \times \exp\{i[2\omega t - \mathbf{k}_2(2\omega)\cdot r]\} + \text{c.c.}, \tag{2}$$

and

$$E_3 = \tfrac{1}{2}A_3 \times \exp\{i[\omega t - \mathbf{k}_3(\omega)\cdot r]\} + \text{c.c.}, \tag{3}$$

where $A_3 \propto A_1^*A_2$. In the event that E_2 is a plane wave, E_3 becomes the conjugate replica of E_1. The field E_3 propagates in the crystal with a wavevector $\mathbf{k}_3(\omega)$, while the polarization source has a wavevector $\mathbf{k}_2(2\omega) - \mathbf{k}_1(\omega)$. Effective transfer of energy from E_2 to E_3 is possible only if $\mathbf{k}_3(\omega) \simeq \mathbf{k}_2(2\omega) - \mathbf{k}_1(\omega)$. This is known as the phase-matching condition. If this condition is not fulfilled, the wavefronts of E_3 radiated at the different regions of the crystal do not add up in phase, and destructive interference will reduce the amplitude of the E_3 so generated. If $\Delta k = |\mathbf{k}_2(2\omega) - \mathbf{k}_1(\omega) - \mathbf{k}_3(\omega)|$ is the mismatch in wavevectors, the crystal length that

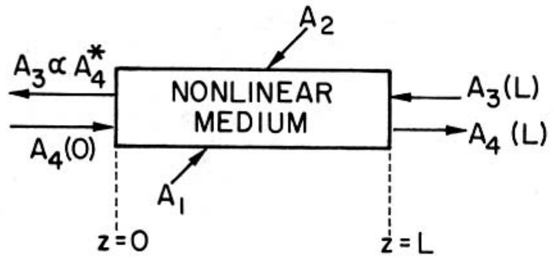

Fig. 1. Four-wave mixing geometry.

can be used such that there is minimal destructive interference effect is about $\pi/\Delta k$,[11] which typically is around 50 μm. Thus phase matching is an important consideration when a long interaction length is wanted to generate a strong E_3. Usually the exact phase-matching condition can be satisfied along one direction only. This causes a severe limitation on the angular divergence of the input wave and hence on the amount of information or distortion that can be conjugated or corrected.

An alternative process, which overcomes the phase-matching requirement, is what is now known as degenerate four-wave mixing. All the optical fields involved in the mixing process are of the same frequency ω; hence the name degenerate. The mixing utilizes an optically induced polarization that is cubic in the electric-field strength. This third-order nonlinear polarization, though of a higher order than that involved in the second-order (three-wave) mixing, is not restricted to asymmetric crystals and can occur in any material, for example, any crystal, liquid, or gas. The geometry of the interaction is shown in Fig. 1. A nonlinear medium, for example, carbon disulfide (CS_2), is illuminated by two intense pump beams, A_1 and A_2, both at frequency ω. These waves are chosen to be counterpropagating plane waves such that their wavevectors $\mathbf{k}_1(\omega)$ and $\mathbf{k}_2(\omega)$ add up to zero. Upon the incidence, along an arbitrary direction, of an arbitrary input wave, A_4, at the same frequency ω but with a wavevector $\mathbf{k}_4(\omega)$, a third-order nonlinear polarization

$$P^{\omega+\omega-\omega} = \chi^{(3)} A_1 A_2 A_4^* \times \exp\{i[(\omega+\omega-\omega)t - (\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_4)\cdot r]\} \quad (4)$$

is formed, which radiates to give a field A_3, also at frequency ω but with a wavevector $\mathbf{k}_3(\omega) = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_4$ that is equal to $-\mathbf{k}_4(\omega)$. $\chi^{(3)}$ is the third-order nonlinear susceptibility of the medium. The backward-going wave A_3, with an amplitude distribution proportional to A_4^*, in turn combines with the two pump waves to induce a third-order nonlinear polarization, varying as $A_1 A_2 A_3^*$, which radiates into A_4. The counterpropagating conjugate pair, A_3 and A_4, are thus coupled together through the nonlinear polarization and are amplified simultaneously. The energy comes from the two pump beams A_1 and A_2. The coupling between A_3 and A_4, which is intermediated via the pump waves A_1 and A_2, served as a feedback mechanism with gain, allowing the oscillation of this conjugate pair if sufficient pump power is furnished.[10] The phase-matching condition is automatically satisfied in this process since $\mathbf{k}_3 = -\mathbf{k}_4$.

The nonlinear medium, used under the condition prescribed above, can be viewed as a mirror, reflecting an arbitrary input wave A_4 to give a backward wave $A_3 \propto A_4^*$. The difference between such a conjugate mirror and a conventional one lies in the fact that the reflected wave is a time-reversed replica and retraces the exact path of the incident wave, whereas the reflected wave from a conventional mirror obeys the conventional law of reflection. Figure 2 uses the example of a wavefront passing through a distorting medium to illustrate the difference between the conjugate mirror and a conventional one. The intensity reflectivity of the conjugate mirror, in the limit of negligible pump-power depletion, is $\tan^2(\kappa L)$. $\kappa \propto \chi^{(3)} A_1 A_2$ depends on the nonlinear properties of the material and the pump power. L is the length of the interaction region. We note that, when $\kappa L > \pi/4$, $\tan \kappa L > 1$ and amplified reflection will occur; and at $\kappa L = \pi/2$, an infinite reflectivity, corresponding to self-oscillation, is predicted.

The first observations of phase conjugation by degenerate four-wave mixing were reported by Stepanov *et al.*[3]

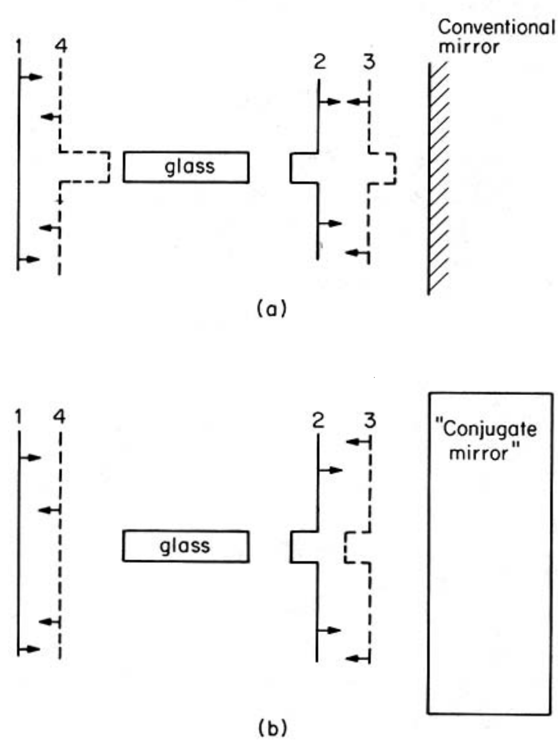

Fig. 2. (a) A plane wave (1) is incident upon a distorting element (a glass cylinder) emerging with a bulge (2). The wave reflected from a conventional mirror (3) traverses the cylinder in reverse, resulting in a doubling of the bulge depth. (b) A conjugate mirror yields a reflected wavefront (3), which is identical to the incident wave (2). The result is a perfect smoothing of the bulge in (4), so that (4) and (1) are identical.

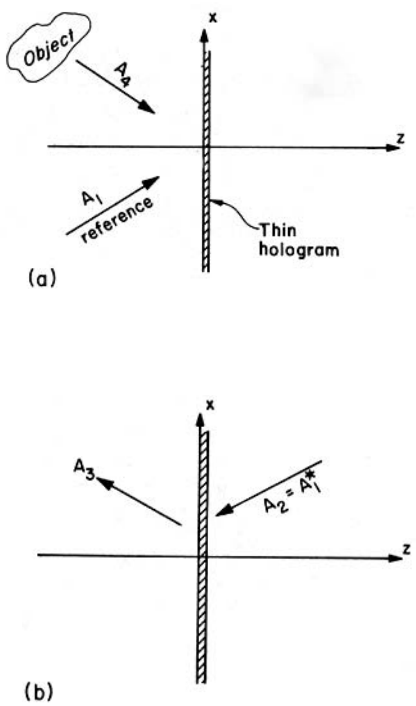

Fig. 3. (a) Formation of the grating, (b) reconstruction.

Amplified phase-conjugation reflection and parametric oscillation were demonstrated by Bloom *et al.*[12] and by Pepper *et al.*[13] The latter experiment used CS_2 as the nonlinear medium, and a Q-switched laser was required to provide the large optical intensities necessary to observe the effects. However, the nonlinear index can be increased considerably if the optical frequency ω is chosen to be close to that of an atomic transition. This has led to the use of low-power laser sources for performing similar experiments in sodium[12,14] and rubidium vapors.[15]

REAL-TIME HOLOGRAPHY

An alternative viewpoint of four-wave mixing has a direct analogy in holography.[3,16] The incident (object) wave A_4 interferes with a (reference) pump beam, say A_1, to form a grating, which then diffracts the other counter-propagating (reconstruction) pump beam, A_2, to give the conjugate image wave, A_3. At the same time, the interference of A_4 and A_2 forms a hologram, which diffracts A_1 to yield A_3. The backward conjugate wave in turn forms a grating with either pump to diffract the other, resulting in the amplification of the incident object wave. Figure 3 shows the hologram recording and the reconstruction processes. Four-wave mixing, however, differs from the conventional holographic experiments in the simultaneity of the grating formation and the reconstruction steps. This makes possible the processing of information in real time. The name real-time holography has frequently been used to refer to this kind of nonlinear optical means of wavefront generation.

Many physical phenomena can be used to perform four-wave phase conjugation. Fundamentally it is necessary that the local optical properties of the medium depend essentially instantaneously on the optical intensity. The modulation of the intensity that is due to the interference of the weak (signal) input wave A_4 with either of the pump waves acts as a grating that scatters the second pump beam, thus creating the conjugate wave. Physical effects used to date include the Kerr effect[17] (change of index of refraction proportional to the intensity), saturable optical absorption,[18,19] and generation of free[4] and localized electrons.[20]

More recently, phase conjugation via real-time holography in barium titanate[21] and BSO crystals[20] was reported. The same results have also been observed in the infrared (10.6 μm) in germanium,[22] SF_6, NH_3, etc. We shall not attempt to give an exhaustive list of the recent work in this field reported by various research laboratories. Most of it was presented at the 1978 International Quantum Electronics Conference in Atlanta and the 1978 annual meeting of the Optical Society of America in San Francisco.

PHASE CONJUGATION IN WAVEGUIDES

The phenomenon of four-wave mixing is a third-order nonlinear optical process and generally needs high laser-beam intensities and long interaction lengths in order to be observable. These conditions become less constraining when the interaction takes advantage of near-resonant atomic transitions, as mentioned above. However, because of the ability of an optical fiber to guide light within a small cross-sectional area and over a long distance, even a small incident light power results in a sufficient intensity inside the fiber for nonresonant nonlinear phenomena and devices to be studied. It was proposed by Yariv *et al.*[23] that phase conjugation by degenerate four-wave mixing can take place efficiently inside optical fibers at low cw laser powers. Figure 4 shows the geometry for such a conjugation scheme. Incident upon the two ends of an optical fiber are two counter-propagating pump fields, E_{p1} and E_{p2}, which are primarily coupled into the lower-order fiber modes. An input probe field, E_i, is also simultaneously incident upon one end ($z = 0$) of the fiber; this probe field carries spatial information and can, in general, excite many fiber modes. The pump and the probe waves originate in a laser that must have a coherence length longer than the fiber to avoid any destructive interference within the entire interaction region. The induced third-order nonlinear polarization in the fiber-core medium radiates a backward-traveling wave, E_r, opposite E_i. It can be shown that each fiber mode in the incident

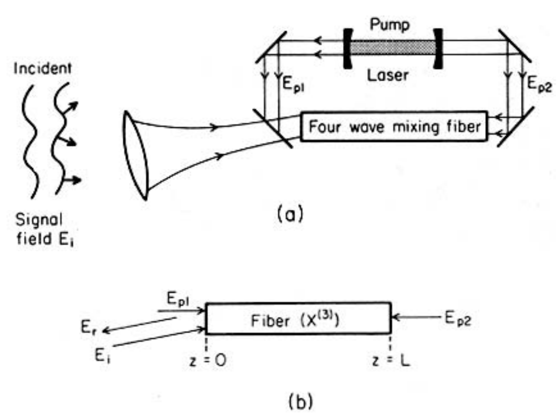

Fig. 4. (a) The basic geometry for four-wave phase conjugation in a fiber.

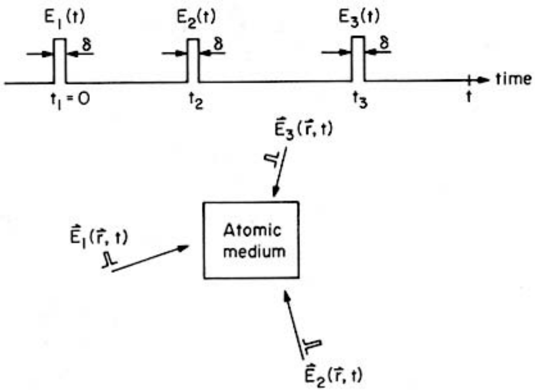

Fig. 5. The geometry for the interaction of the three-pulse sequence with the atomic medium.

probe wave couples most strongly, via the nonlinear polarization, to the same mode of the backward-going wave, which, on arriving at the $z = 0$ end of the fiber, as indicated in the figure, is a time-reversed replica of the input probe wave. This was recently verified experimentally by AuYeung *et al.*,[24] when a 3-m-long CS_2-filled 4-μm i.d. optical fiber was used with a cw laser power of only several milliwatts. Phase conjugation in a light pipe was demonstrated by Jensen and Hellwarth.[25] Their experiment utilized an 80-cm long, CS_2-filled, 0.4-mm i.d. glass tube. A *Q*-switched ruby laser was used to provide 35-kW peak power to each pump beam. They showed that high-fidelity wavefront replication can be achieved even if the pump beams excite many waveguide modes.

PHASE CONJUGATION USING PHOTON ECHOES

All the optical mixing schemes for phase conjugation require the simultaneous illumination of a medium with several intense optical beams. There is, however, another phase reversal technique, which uses the finite "memory" time of the atoms in the "mixing" medium. This technique does not require the optical fields to be present simultaneously. It was first suggested by Heer and McManamon[26] and later by Shiren[27] that the phenomenon of photon echoes can be used as a wavefront conjugation technique. Consider an ensemble of two level atoms, with resonant frequency ω, subjected to three noncoincident pulses of optical radiation of the same frequency ω but traveling along arbitrary directions with different polarizations. These three fields, E_1, E_2, and E_3, have wavevectors $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$, respectively. The arrangement is sketched in Figure 5. The pulse widths δ and the time intervals between the pulses are smaller than the atomic relaxation time. In the limit of small "area" pulses ($\mu E\delta/\hbar << 1$), the interaction between the light pulses and the medium can be described by using a perturbation approach[28] analogous to that of the optical mixing and the real-time holography. The incidence of the first pulse $E_1(t)$ at time t_1 perturbs the atomic system and induces a dipole moment, which decays exponentially in time at a rate of $1/T_2$. The arrival of the second pulse $E_2(t)$ at time t_2, with $t_2 - t_1 << T_2$, modulates the population difference of the two-level system about its thermal-equilibrium value. This population grating decays exponentially in time at a rate $1/T_1$. It diffracts the third pulse to give the conjugate wave, which, with an amplitude proportional to $E_1^* E_2 E_3$, again decays exponentially at the rate $1/T_2$. When the response is summed over the inhomogeneous distribution of the resonant frequencies of the atoms, the conjugate wave appears in the form of a short pulse occurring at the time $t_3 + t_2 - t_1$. If the pulses E_2 and E_3 are parallel to each other as well as to the pulse E_1, i.e., $\mathbf{k}_1 = \mathbf{k}_2 = \mathbf{k}_3$, then a forward echo parallel to $\mathbf{k}_1$ will be induced. If $\mathbf{k}_2$ is chosen to be equal to $-\mathbf{k}_3$, then it is a backward echo. In the special case in which E_3 is absent, an echo is formed at time $2t_2 - t_1$, with an amplitude proportional to $E_1^* E_2{}^2$ and a wavevector equal to $2\mathbf{k}_2 - \mathbf{k}_1$. The entire process can be viewed as one whereby a "hologram" is written into the atomic medium by the pulses $E_1(t)$ and $E_2(t)$. An interrogation pulse at t_3 reconstructs the hologram to generate the conjugate pulse. The atomic state retains a memory for a time T_2 (or T_1) and hence the pulses need not be simultaneously present. The experimental demonstration of phase conjugation by photon echoes was recently achieved in sodium vapor by Griffen and Heer.[29]

APPLICATIONS

The most frequently quoted example for the application of phase conjugation is the correction of aberrations that arise when an optical wave passes through a phase-distorting medium.[30] Consider the example of sending spatial information that is encoded on an optical carrier wave. After propagating through a communication channel, such as a multimode optical fiber, the information is scrambled because of undesired phase distortion. In the case of a multimode fiber, this arises from the fact that each fiber mode propagates with a different phase velocity. If a conjugate wave is generated at this point to reverse the phases and then allowed to propagate through another similar section of the fiber, the phase acquired by each fiber mode is canceled, and the information is thus recovered. Figure 6 shows the schematic for such an information-restoration technique.

We have mentioned that a nonlinear material, when used in the four-wave-mixing geometry, can be treated as a conjugate mirror. If such a mirror is

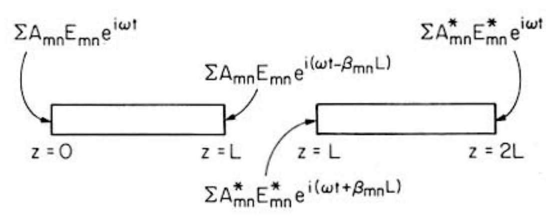

Fig. 6. Compensation for image distortion by modal dispersion in a dielectric waveguide using phase conjugation.

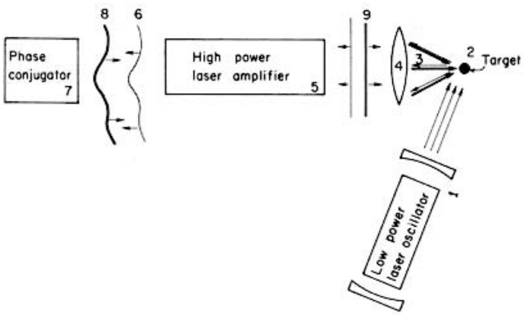

Fig. 7. A configuration designed to focus the output of a high-power pulsed laser on a target.

used to replace one of the two conventional mirrors that form a laser cavity, it has been observed that there is no stability condition necessary for oscillation to occur. It also compensates for phase distortions contributed by any poor intracavity optical components, which otherwise will degrade the optical quality of the laser output beam.

Four-wave mixing has been shown to be analogous to holography. Many applications normally associated with holography can thus be performed via nonlinear optical mixing in real time. For example, it has been proposed to use four-wave mixing to perform spatial convolution and correlation of optical fields.[31]

One of the most important potential applications of phase conjugation involves the correction of aberrations in laser amplifiers and in other dynamic optical systems in real time. One method of accomplishing this aim was demonstrated by the experiment of Zeldovich *et al.*[5] Another possible method is illustrated in Fig. 7. The figure shows a configuration designed to focus the output of a high-power pulsed-laser amplifier on a very small target.

If the output of the amplifier were to be focused directly on the target, the distortion of the amplifier medium during the pump pulse would make it impossible to obtain a diffraction-limited spot size, thus reducing the available intensity.

In the compensation scheme the target (2) is illuminated by the output of a low-power laser oscillator (1) (cw or pulsed). The reflected light (3) is collimated by lens (4) and traverses the amplifier (5) in reverse (away from the target). The distorted wavefront (6) is incident upon the conjugator (7) and is reflected as a conjugate wave (8). The wave retraverses the amplifier (5), emerging as a planar wave (9), which is focused on the target as a diffraction-limited spot.

A most important attribute of this scheme is its ability to track a moving target, provided that the distance traversed during the round-trip time from the target to the conjugator is small compared to the spot size. As a matter of fact, if the gain of the amplifying medium is sufficiently high and/or the reflection of the conjugator is strong, then an oscillation mode can exist in which the target serves as one reflector while the conjugate reflector completes the resonator. In this mode of oscillation[10] the beam autotracks the moving target. The low-power illuminating laser (1 in Fig. 7) is not needed in this mode of oscillation.

Other applications include optical filtering,[32] compensation for group-velocity dispersion in a communication channel,[33] and optical gating. Active research is still going on in this relatively new area. Many promising and exciting applications have yet to be explored.

This research was supported by the U.S. Army Research Office, Durham, North Carolina.

REFERENCES

1. R. J. Collier, C. B. Burckhardt, and L. H. Lin, in *Optical Holography* (Academic, New York, 1971), p. 368.
2. J. Opt. Soc. Am **67** (March, 1977) (special issue on adaptive optics).
3. B. I. Stepanov, E. V. Ivakin, and A. S. Rubanov, Sov. Phys. Dokl. **16**, 46, 1971.
4. J. P. Woerdman, Opt. Commun. **2**, 212 (1970).
5. B. Ya. Zel'dovich, V. I. Popovichev, V. V. Ragul'skii, and F. S. Faisullov, Sov. Phys. JETP **15**, 109 (1972); O. Yu. Nosach, V. I. Popovichev, V. V. Ragul'skii, and F. S. Faisullov, Sov. Phys. JETP **16**, 435 (1972).
6. A. Yariv, Appl. Phys. Lett. **28**, 88, 1976.
7. A. Yariv, J. Opt. Soc. Am. **66**, 301, 1976.
8. P. V. Avizonis, F. A. Hopf, W. D. Bamberger, S. F. Jacobs, A. Tomita, and K. H. Womack, Appl. Phys. Lett. **31**, 435 (1977).
9. R. W. Hellwarth, J. Opt. Soc. Am. **67**, 1 (1977).
10. A. Yariv and D. M. Pepper, Opt. Lett. **1**, 16 (1977).
11. A. Yariv, *Quantum Electronics*, 2nd ed. (Wiley, New York, 1975).
12. D. M. Bloom, P. F. Liao, and N. P. Economou, Opt. Lett. **2**, 58 (1978).
13. D. M. Pepper, D. Fekete, and A. Yariv, Appl. Phys. Lett. **33**, 41 (1978).
14. P. F. Liao, D. M. Bloom, and N. P. Economou, Appl. Phys. Lett. **32**, 813 (1978).
15. D. Grischkowsky, N. S. Shiren, and R. J. Bennett, Appl. Phys. Lett. **33**, 805 (1978).
16. A. Yariv, Opt. Commun. **25**, 23 (1978).
17. D. M. Bloom and G. E. Bjorklund, Appl. Phys. Lett. **31**, 592 (1977).
18. R. L. Abrams and R. C. Lind, Opt. Lett. **2**, 94 (1978).
19. P. F. Liao and D. M. Bloom, Opt. Lett. **3**, 4 (1978).
20. J. P. Huignard, J. P. Herrian, P. Aubourg, and E. Spitz, Opt. Lett. **4**, 21 (1979).
21. J. Feinberg, D. Heiman, and R. W. Hellwarth, J. Opt. Soc. Am. **68**, 1367A (1978).
22. E. E. Bergmann, I. J. Bigio, B. J. Feldman, and R. A. Fisher, Opt. Lett. **3**, 82 (1978).
23. A. Yariv, J. AuYeung, D. Fekete, and D. M. Pepper, Appl. Phys. Lett. **32**, 635 (1978).
24. J. AuYeung, D. Fekete, D. M. Pepper, A. Yariv, and R. Jain, Opt. Lett. **4**, 42 (1979).
25. S. M. Jensen and R. W. Hellwarth, Appl. Phys. Lett. **33**, 404 (1978).
26. C. V. Heer and P. F. McManamon, Opt. Commun. **23**, 49 (1977).
27. N. S. Shiren, Appl. Phys. Lett. **33**, 299 (1978).
28. A. Yariv and J. AuYeung, to appear in IEEE J. Quantum Electron. **QE-15**, April 1979.
29. N. C. Griffen and C. V. Heer, Appl. Phys. Lett. **33**, 865 (1978).
30. A. Yariv, IEEE J. Quantum Electron. **QE-14**, 650 (1978).
31. D. M. Pepper, J. AuYeung, D. Fekete, and A. Yariv, Opt. Lett. **3**, 7 (1978).
32. D. M. Pepper and R. L. Abrams, Opt. Lett. **3**, 212 (1978).
33. A. Yariv, D. Fekete and D. M. Pepper, Opt. Lett. **4**, 52 (1979).