OPTICAL CHAOS

By Peter W. Milonni, Jay R. Ackerhalt, and Mei-Li Shih

Notions of order and chaos go back a long way. The Greeks held that all motion could be decomposed into "perfect" circular motions, and this belief led to the theory of planetary epicycles. We might phrase the Platonic ideal this way: all motion is *quasiperiodic*, meaning the Fourier transform of any coordinate consists of sharp spikes (Fig. 1). Poincaré near the turn of the century, was perhaps the first person to realize that there are (bounded) motions whose spectra do not have this form. Such systems have a broadband, continuous component in their spectra, as shown in Fig. 2.

Spectra of the type illustrated in Fig. 2 are associated with turbulent motion. Imagine two corks floating near each other in some fluid, and suppose the fluid flow is laminar (i.e., smooth and orderly). As time evolves the positions of the corks are correlated and their separation might grow linearly in time. But if the flow is turbulent, the corks separate rapidly—typically exponentially with time—and their locations depend very sensitively on where they started out. A system is called *chaotic* if it has this property of very sensitive dependence on initial conditions.

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Chaotic motion

Chaotic motion is non-quasiperiodic, having a spectrum like that sketched in Fig. 2. Over the past few years, many physical systems have been found to evolve chaotically. Among recent developments two are especially noteworthy. One is the recognition that chaos may appear in systems described by relatively simple rules of evolution. The other is the discovery of some prevalent, universal ways by which systems may make the transition from orderly, quasiperiodic behavior to chaos. These same routes to chaos have been observed in systems as diverse as lasers,[1] fluid flows,[2] semiconductor devices,[3] and bouncing balls,[4] to name but a few. The one thing these systems have in common is that they are nonlinear, for the kind of chaos we are talking about cannot appear in purely linear systems.

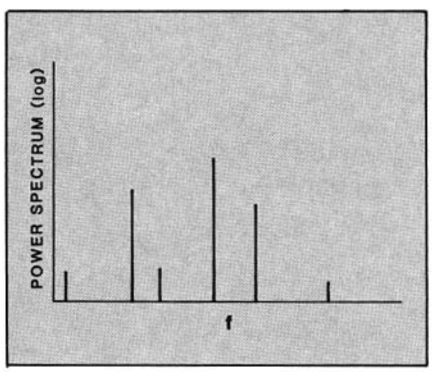

FIGURE 1. Discrete spectrum of a quasiperiodic system.

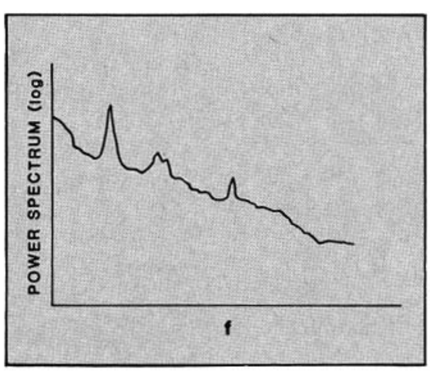

FIGURE 2. Continuous spectrum of a chaotic system.

The remarkable thing is that such chaos is the result of deterministic rules of evolution, with no stochastic elements in either the equations of motion or the input state. How "random" can such chaos be? It turns out that it can be as random as the sequence of heads and tails produced in a game of coin tossing.[5] Although the system is fully deterministic, sensitivity to initial conditions is so strong that the output would be judged to have a quality of randomness.

A dissipative chaotic system stretches the distance between initially close points (like the corks in our example), but at the same time manages to keep the motion bounded within a certain portion, called an attractor, of the space of possible states. It does this by a combination of stretching and folding that produces a self-similar or "fractal" object in state space, a so-called strange attractor.

We will describe what appear to be the three most important routes to chaos, and illustrate them with some results obtained with lasers.

The different scenarios for the transition from order to chaos assume that, as some parameter or "knob" of a system is varied, a sequence of "bifurcations" occurs that eventually results in chaotic motion. In a Hopf bifurcation, for instance, a system might go from a time-independent equilibrium state to an oscillatory state.

Perhaps the oldest scenario for the transition to chaos is that proposed by Landau in 1944. In the Landau scenario chaos is viewed as the result of an infinite sequence of Hopf bifurcations or, in other words, as the result of a proliferation of frequencies generated by nonlinear couplings. But we now know that a system with a spectrum made up of discrete spikes (Fig. 1), however many, cannot be chaotic in the sense of extreme sensitivity to initial conditions. That is, the Landau scenario does not describe a transition to chaos per se.

In 1971 Ruelle and Takens argued that the Landau scenario is even unlikely in that, after a few Hopf bifurcations, a system will tend to become chaotic as the knob is varied further. Various systems have been observed to become chaotic after the appearance of just two incommensurate frequencies. Figure 3, for instance, shows experimental spectra of Weiss et al.[6] obtained from the output of a 3.39 μm HeNe laser. The different traces were obtained by tilting one of the laser mirrors. In the lower trace there is a single frequency and its second harmonic. In the middle trace there are two basic frequencies, f_1 and f_2, as well as linear combinations of these two frequencies. Upon further mirror tilt this two-frequency motion gives way to chaos, with the characteristic broadband spectrum shown in the upper trace of the figure.

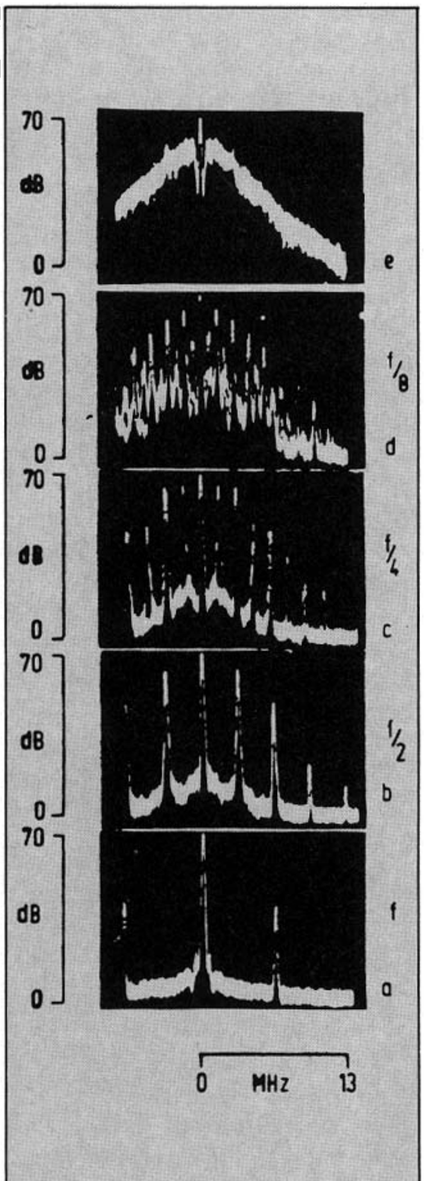

FIGURE 3. Two-frequency route to chaos in a HeNe laser. (Courtesy of C.O. Weiss)

The period doubling route

In the period doubling route to chaos, there is a sequence of period doubling (or pitchfork) bifurcations as a system parameter is varied, and chaos results after period doubling has occurred ad infinitum. At each bifurcation a frequency f in the spectrum leads to the subharmonic frequency f/2 as a system parameter is swept. Feigenbaum has discovered quantitative features of the period doubling sequence that apply to virtually all systems undergoing this route to chaos.

The period doubling route has been observed in many different physical systems. In 1982 Arecchi et al.[7] reported the first observation and characterization of chaos in a laser system. They modulated the cavity loss of a CO_2 laser and observed a period doubling to chaos as the modulation frequency was varied. Figure 4 shows spectra recorded by Weiss et al.[6] for a different range of mirror tilts than for Fig. 3. Three period doubling bifurcations are evident in going from (a) to (d), and in (e) we see again the broadband wash characteristic of chaos. The presence of experimental "noise" usually prevents the observation of more than three or four period doubling bifurcations, although a very large number of period doublings can be seen in computer "experiments."

In the intermittency route to chaos

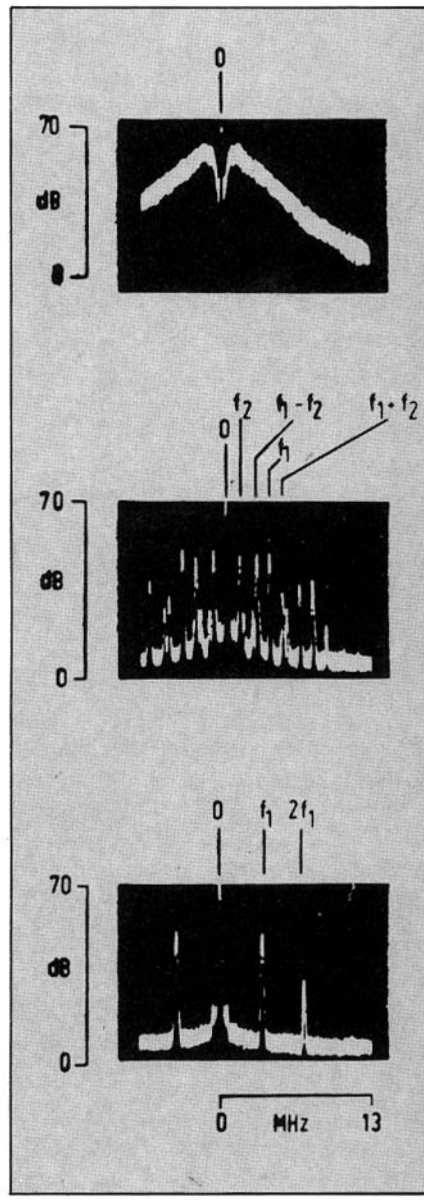

FIGURE 4. Period doubling to chaos in a HeNe laser as one of the mirrors is tilted. (Courtesy of C.O. Weiss)

described by Pomeau and Manneville in 1980, chaos develops via the appearance of increasingly frequent, irregular bursts interrupting otherwise orderly oscillations. There are various types of intermittency, each corresponding to a different way by which stable motion becomes unstable. Figure 5 shows experimental results obtained by Weiss, et al.[6] for intermittency in their HeNe experiments.

We should also mention the work of Abraham et al.[8] on chaos in single-mode, Doppler-broadened systems. In particular, experiments in the operating regime of the "Casperson instability"[9] provided clear evidence of the three routes to chaos just described. We and others have found these same routes in simplified numerical simulations involving as many as 200 coupled differential equations, each corresponding to a different velocity group.

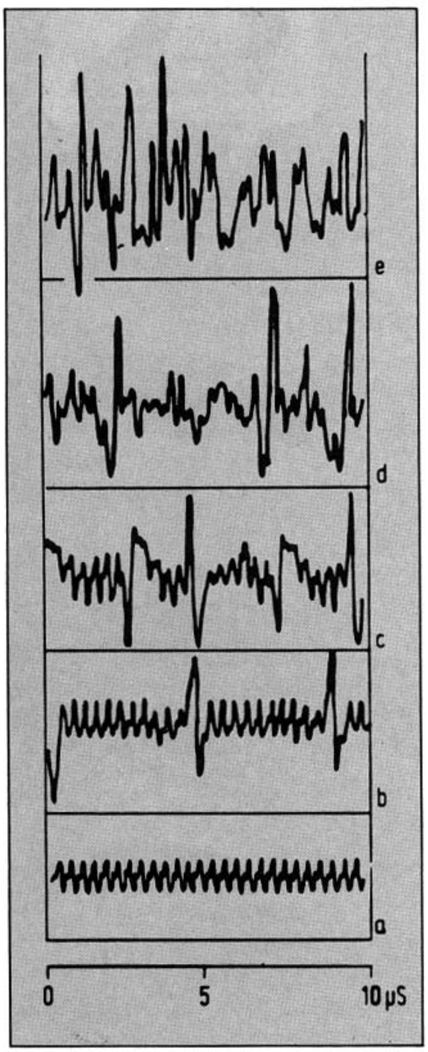

FIGURE 5. Intermittency route to chaos in a HeNe laser. (Courtesy of C.O. Weiss)

Chaos has also been observed in passive optically bistable systems, as first predicted by Ikeda.[10] An early experiment of Gibbs et al.[11] employed a "hybrid" device in which a delay time appearing in the theory was introduced electronically by delayed feedback. Later Nakatsuka et al. [12] also observed period doubling to chaos in an "all-optical" system employing a single-mode fiber.

Classical chaos

It should be clear that much progress has been made in the study of chaos at the classical (macroscopic) level. Present work in optical chaos extends the earlier studies to a wider range of systems, and includes improvements of earlier theoretical models as well as efforts to measure such things as Lyapunov exponents and dimensions of strange attractors. It is not clear whether any applications will come out of all this, but these studies have genuinely advanced our understanding of optical instabilities.

There are many interesting questions connected with the possibility of chaos in a single atom or molecule exchanging energy with a field. Such studies deal with two levels of chaos— the deterministic chaos of classical systems, as discussed above, and the fundamental sort of chaos inherent at the quantum level. Theoretical difficulties connected with the quantization of classically chaotic systems were recognized by Einstein in 1917 in connection with the old quantum theory, which employed certain (Bohr-Wilson-Sommerfeld) rules for the quantization of quasiperiodic classical systems. Einstein addressed the question of how to quantize non-quasiperiodic classical systems. The issue became largely irrelevant with the advent of the Schrödinger equation, which is free of any assumptions about separability of the classical motion. The question now is how classical chaos might manifest itself quantum mechanically. Is the quantum behavior of classically chaotic systems much different from that of classically quasiperiodic systems?

Quantum systems with discrete energy levels are quasiperiodic and so cannot exhibit the extreme sensitivity to initial conditions that is the hallmark of chaos in classical systems, i.e., systems for which quantum effects are completely negligible. The quantum-mechanical wave function evolves in an orderly, predictable way, although it can only provide probabilistic information about the system. A chaotic classical system, on the other hand, is in principle deterministic, but in practical terms we cannot make detailed, long-term predictions about its behavior. In this sense quantum systems evolve in a more orderly fashion than their classical counterparts.[13]

One aspect of the quest for "quantum chaos" concerns the growth of energy of a system driven by an external force, such as an atom in the field of a laser. In some classical models of periodically driven systems, the energy grows diffusively, proportional on average to the time. When the same models are described quantum mechanically, however, the energy grows diffusively at first but the growth eventually ceases.[14] This quantum suppression of classical diffusive behavior can be related mathematically to the Anderson localization of a particle in a one-dimensional lattice with random site energies.[15] and provides another indication of the more orderly time evolution of quantum systems.

Order vs. chaos

Chaos in an idealized classical model offers a possible explanation of certain features observed in experiments on the multiple-photon excitation of molecular vibrations.[16] However, questions of order versus chaos are important even to our understanding of how the hydrogen atom interacts with electromagnetic radiation. Beginning with the work of Bayfield and Koch[17] in 1974, experimenters over the past decade have studied the excitation and ionization of highly excited hydrogen atoms in a microwave field. The plethora of states involved has made quantum-mechanical analyses of these experiments difficult, although classical computations have shown rather impressive agreement with experiment.[18] The classical dynamics is found to be chaotic, with extreme sensitivity of the electron motion to the initial conditions. And so the old question of "quantum chaos" is now at the forefront of efforts to understand the interaction of light with the very simplest of atoms.[19]

Although various scientists in the past, including Maxwell and Born, have emphasized the degree of complexity possible in relatively simple systems, it has taken a long time for this reality to become part of the general world view.

As Feynman says, "Unaware of the scope of simple equations, man has often concluded that nothing short of God, not mere equations, is required to explain the complexities of the world."[20] It is the computer, more than anything else, that is changing this misconception and providing us with a fresh way of thinking about turbulent behavior in optics and other branches of science.

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