Controlling Chaotic Lasers

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Do lasers really exhibit chaos? Irregular intensity fluctuations were observed even in the earliest days of lasers. For example, the "spiking" fluctuations present in the output of a ruby laser were difficult to miss. All fluctuation phenomena observed were then commonly labeled "noise." Since that time, however, it has become evident that apparently random fluctuations can also occur in totally deterministic systems—those that are modeled by systems of equations containing no sources of noise. Such behavior is commonly called chaos.

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The purpose of this article is to describe the advances made in the identification of chaotic behavior in lasers and to focus on the specific theme of controlling chaotic lasers. We will first briefly review the origins of the basic concepts of chaos and nonlinear dynamics in a general context and see how beautifully the history of chaotic lasers is intertwined with that of meteorology and fluid dynamics, the fields in which the crucial role of chaos was first recognized.

One of the most important realizations of the last 30 years has been that chaotic behavior is commonplace in physical, chemical, and biological systems.[1] Most scientists and engineers have begun to recognize this inescapable fact. The advent of computers has been responsible for this awakening, not just in meteorology, but in every branch of science, from astronomy to zoology. A new branch of mathematics, dynamical systems theory, has developed rapidly and now forms the universal mathematical language for the description of chaotic systems in science and engineering. Nonlinear dynamics is the discipline that includes experimental and theoretical investigations of chaos and instabilities.

Chaos and the Weather

John Von Neumann, the father of modern computers, dreamed that one day we would be able to predict accurately, and perhaps even control, weather patterns around the globe.[2] In his book *Infinite in All Directions*, physicist Freeman Dyson[3] discusses the views of Von Neumann, who had the "reputation of being the cleverest man in the world." Von Neumann hoped that computers would allow us to "divide the phenomena of meteorology cleanly into two categories, the stable and the unstable. The unstable phenomena are those that are upset by small disturbances, the stable phenomena are those that are resilient to small disturbances . . . . All processes that are stable we shall predict. All processes that are unstable we shall control."

Dyson remarks that few people took Von Neumann's dream seriously, including meteorologists. In fact, studies on the problem of convection of a fluid heated from below (a highly simplified model of the Earth's atmosphere) by Edward Lorenz, an MIT meteorologist, seemed to indicate that Von Neumann's dream could never be realized. Lorenz's numerical computations[4] revealed a totally new aspect of behavior in this dynamical system; large, irregular fluctuations appeared to originate from an innocuous looking set of three-coupled, nonlinear ordinary differential equations without any sources of noise or fluctuations included in them.

Even more surprising was the incredible sensitivity of a solution of these equations to a small difference in initial conditions. Lorenz found that very slightly different initial conditions resulted in an exponential divergence of solutions. Lorenz engraved this aspect of chaotic dynamics in our minds through the title of a talk he gave in 1972, "Predictability: Does the Flap of a Butterfly's Wings in Brazil set off a Tornado in Texas?"[1(d)] Indeed, Dyson comments that "a chaotic motion is generally neither predictable nor controllable. It is unpredictable because a small disturbance will produce exponentially growing perturbation of the motion. It is uncontrollable because small disturbances lead to other chaotic motions and not to any stable and predictable alternative. Von Neumann's mistake was to imagine that every unstable motion could be nudged into a stable motion by small pushes and pulls applied at the right places."

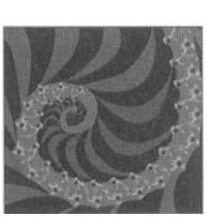
In the few years since Dyson wrote about the failure of Von Neumann's dream, there has, in fact, been significant progress toward its realization; reasonably accurate predictions can be made for chaotic motion over fairly long periods of time,[5] and the control of chaotic motion has been demonstrated for some "simple" chaotic systems, including lasers.

Chaotic Lasers

The connection between chaotic dynamics and laser instabilities was not made until Hermann Haken, in a short paper,[6] remarked on a beautiful similarity that he had discovered between the equations for a fluid studied by Lorenz a dozen years earlier, and the semi-classical equations that describe the operation of a single-mode laser. He found that the three equations of motion for the electric field of the laser, the polarization of the active medium, and the population inversion were identical in form to the Lorenz equations after appropriate transformations of the variables. These equations contained no noise sources, yet their similarity to the Lorenz equations implied that a laser should display irregular deterministic fluctuations in certain parameter regimes.

A search for laser chaos ensued, and several groups around the world searched for the right laser system to display Lorenz chaos.[7] This was not as simple as it may seem. A single-mode laser system had to be found where the decay rates of the polarization and population inversion of the active medium, as well as that of the electric field were of the same order of magnitude, for a valid comparison with the Haken-Lorenz equations. In most laser systems (He-Ne, CO_2, semiconductor, Nd:YAG, etc.), the polarization decay rate is much greater than the inversion and field decay rates, resulting in the effective reduction of the three variable system of equations to two dynamical variables. Chaos cannot occur in a two variable system of equations; a mathematical theorem tells us that we can only have stable or periodic dynamics in a two-dimensional system. We thus have to find a single-mode laser in which all three decay rates are comparable to see this type of chaotic behavior. Finally, a rather exotic laser system, the far-infrared ammonia laser, was settled upon, which had the right features to display Lorenz-like chaos.[8,9]

How can we tell if the observed intensity fluctuations in such a laser are really a signature of chaotic behavior? One of the most straightforward approaches to this question is to examine the system behavior while varying one of the parameters. If a sequence of behavior, or route to chaos, is found that has been identified from the study of deterministic model equations for the system, one may be reasonably sure that chaotic behavior has been observed. Extensive experiments and numerical modeling by several groups have now established that chaotic behavior is indeed displayed by the ammonia laser.



The reader may ask if laser chaos is restricted to exotic systems such as the ammonia laser. Many "garden variety" laser systems can exhibit chaotic behavior if they operate in multiple longitudinal or transverse modes; once again, these additional modes provide the three or more degrees of freedom necessary for the system to be chaotic. If the modes are nonlinearly coupled to each other, chaos can result. For example, nonlinear mode-coupling through four-wave-mixing in the active medium may generate chaotic intensity fluctuations of individual modes in a multimode dye laser.[10] External feedback often leads to chaos in semiconductor lasers,[11] a matter of great practical concern. During the 1980s, there were also several observations of chaotic behavior in single-mode lasers with modulated losses and pumps. Arecchi and colleagues demonstrated chaos in a loss modulated CO_2 laser,[12] while Winful and colleagues[13] showed that under certain conditions a semiconductor laser could be driven chaotic by periodic modulation of its injection current. The periodic modulation effectively provides the third degree of freedom necessary to observe chaos in these laser systems. One of the most interesting examples of laser chaos was discovered by Tom Baer (then at Spectra-Physics), who studied the generation of green light from a diode laser pumped Nd:YAG laser with an intracavity KTP crystal.[14]

The Green Problem

Baer found that though the Nd:YAG laser operated in a stable steady state without the intracavity crystal, large irregular intensity fluctuations were sometimes observed when the intracavity KTP crystal was used to generate green light from the system (Fig. 1). Baer noted that this behavior occurred when the laser operated in three or more longitudinal modes. He hypothesized that sum-frequency generation in the KTP crystal could provide mode-mode coupling that would destabilize the laser. This was not a desirable situation for proposed practical applications of the system, in optical disk readers, for example. The unstable behavior of this system soon came to be known as the "green problem."

The chaotic nature of the green laser was investigated in some detail and connected to the destabilization of relaxation oscillations.[15] Relaxation oscillations are always present in a laser; they are the result of power exchange between the atoms of the active medium and the electric field in the cavity and are normally very small in amplitude. It was found that the nonlinear coupling of the modes through sum-frequency generation resulted in the destabilization of relaxation oscillations in the green laser system. A reasonably accurate model was developed for the system, that could predict many aspects of system behavior, both chaotic and non-chaotic.

As may be expected, several methods were proposed and implemented to get rid of the fluctuations. These methods consisted of system modifications such as restricting the laser to operate in two orthogonally polarized modes by adding wave plates to the laser cavity[16,17] or proper orientation of the YAG and KTP crystals.[18] These are typical examples of what has been the traditional reaction of scientists and engineers when faced with irregular fluctuations in a laser system—redesign the system so that it is inherently stable or try to find a parameter regime of the system where chaos does not exist. A departure from this traditional mindset required a radically new perspective and approach toward grappling with chaotic systems.

Controlling Chaos

In the spring of 1990, Ott, Grebogi, and Yorke (OGY) of the University of Maryland introduced such a new perspective in a seminal paper[19] entitled "Controlling Chaos." "Control" refers to achieving periodic or stable output from a chaotic system without changing the parameters of the system, or the system itself, in a permanent way; only small time-dependent perturbations about the ambient parameter values are allowed. OGY observed that when irregular, chaotic behavior is observed, we often do not have available a detailed mathematical model for the dynamical system that will accurately describe its behavior over a wide range of operating parameters. If we want to develop techniques for control of such chaotic systems, two crucial questions arise.

- Can we develop a dynamical control strategy based primarily on experimental measurements made on the system, without trying to build a mathematical model that is globally accurate?
- Can we control the system without making large changes in parameters or variables?

OGY showed that both these goals can be achieved, at least for some chaotic systems. A chaotic system can be controlled with small, judiciously chosen changes to parame-

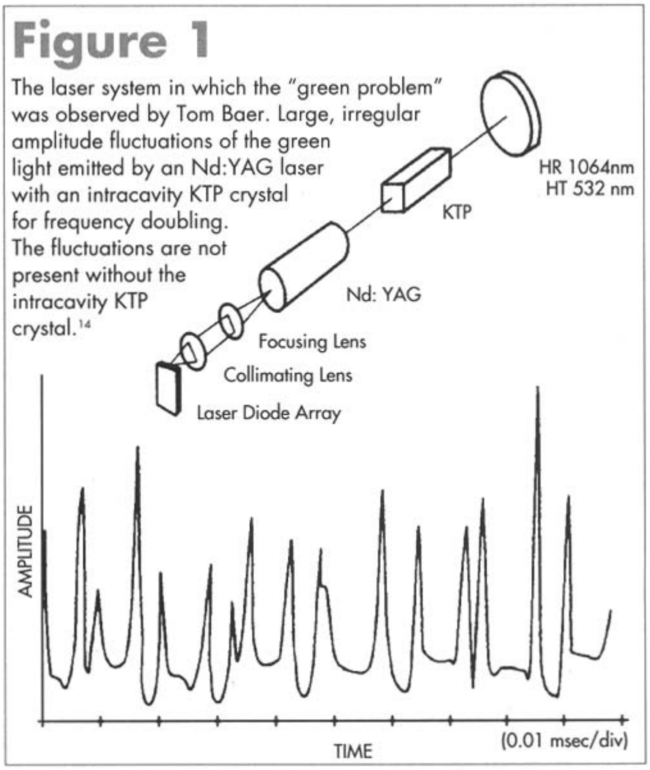
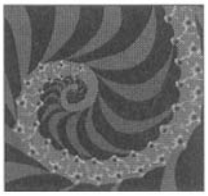


Figure 1
The laser system in which the "green problem" was observed by Tom Baer. Large, irregular amplitude fluctuations of the green light emitted by an Nd:YAG laser with an intracavity KTP crystal for frequency doubling. The fluctuations are not present without the intracavity KTP crystal.[14]



ters made on the basis of observations of a system variable, such as the fluctuating output intensity of a laser. It is the very sensitivity of a chaotic system to small perturbations that allows us to control it with such corrective changes.

The OGY algorithm for chaos control was based on the observation that a chaotic attractor—the geometrical object toward which a system's trajectory in phase space converges—typically has a large number of unstable periodic orbits embedded in it. The system visits the neighborhood of these unstable periodic orbits from time to time; what is needed for control is a technique to nudge the system back to a periodic orbit when the system shows its inherent tendency to depart from it. The basic elements of the OGY algorithm are simple. Even a nonlinear system can be described by a linear approximation, once it approaches close enough to a periodic state or fixed point (for example, an unstable steady state). By observing the dynamics of the system in the neighborhood of the fixed point or periodic state, the direction and amount of instability can be determined. We can then use that information to keep the system near the fixed point or periodic orbit.

To make this point clear, imagine trying to balance a ball at the center of a saddle. The saddle surface is unstable in the direction of convexity; the ball will fall off along the sides of the saddle. The amount of instability, or how fast the ball falls off, is determined by the curvature of the saddle. In the other direction, the saddle is stable; the ball returns toward the center if displaced along the ridge of the saddle. The OGY algorithm tells us essentially how to move a saddle under the ball so as to keep it balanced at the center. Once we know the curvature of the saddle in the unstable direction, we can balance the ball at the center by making observations of the position of the ball from time to time. If control is initiated when the ball is sufficiently close to the center, we can maintain control in a small neighborhood of the center with only small corrective motions.

The OGY algorithm was implemented in a beautiful experiment in late 1990. Ditto, Rauseo, and Spano showed how the OGY algorithm could be applied to stabilize the irregular motion of a mechanical system—a tinsel-like ribbon of magnetoelastic material that swayed chaotically in an applied alternating magnetic field.[20,21] The OGY method and related theoretical and experimental developments in physics, chemistry, and biology have recently been reviewed by Shinbrot *et al.*[22]

The dynamical control of chaotic systems offers several possibilites that are difficult to achieve with the traditional approach in which we adjust system parameters to be in a periodic or stable regime. First, it is possible to switch between two or more periodic waveforms rapidly with dynamical control. Second, it is possible to stabilize complex waveforms that may only occur over a very small parameter range for the system without control. Finally, with active feedback we can extend the range of system parameters over which a periodic orbit or steady state can be maintained.

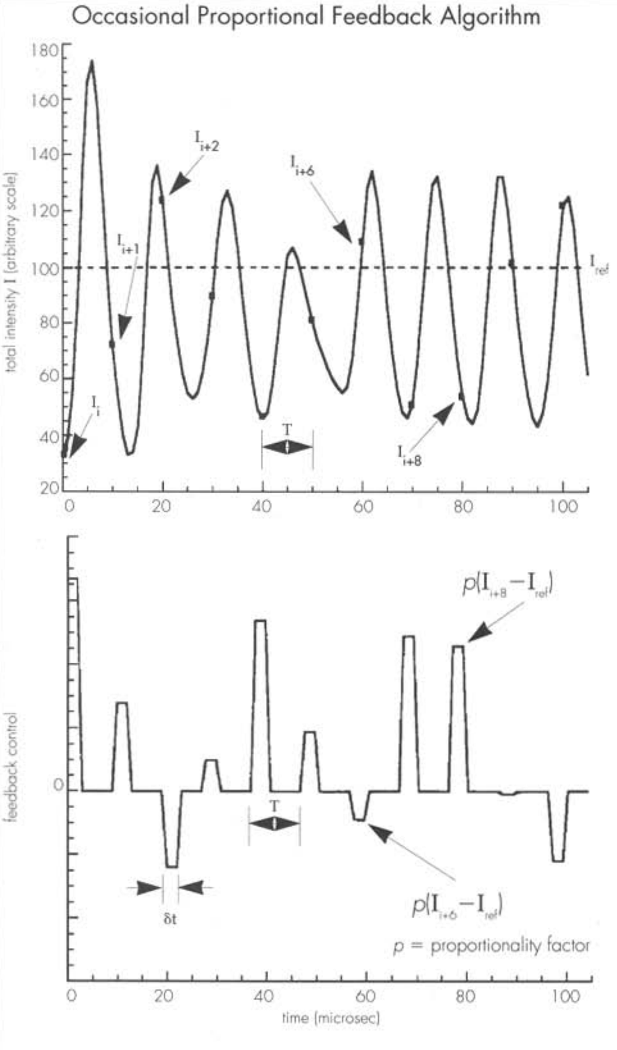


Figure 2

The occasional proportional feedback (OPF) algorithm used for control of the chaotic laser. The four parameters of the control circuit, T, dt, p and I_{ref} are shown.

Dynamical Control of a Chaotic Laser

It was clear to us at Georgia Tech, soon after publication of the OGY paper, that it would be of great interest to try and apply these new techniques to the chaotic green laser. There was the purely scientific motivation: Could we demonstrate control of a chaotic laser in an experiment and stabilize several different periodic waveforms for the same laser parameters? There was also the practical motivation: Could such control techniques be used to stabilize chaotic lasers without having to redesign the system?

It was at this point that one of us (Roy) happened to learn that Earle Hunt (Ohio University) had developed an analog circuit to stabilize periodic waveforms generated by a chaotic diode resonator circuit. Hunt used a variant of the OGY approach, which he called occasional proportional feedback (OPF).[23,24] The name arose from the fact that the feedback consisted of a series of perturbations of limited duration dt ("kicks") delivered to the input drive signal at periodic intervals (T) in proportion to the difference of the chaotic output signal from a reference value. The OPF technique seemed perfectly suited for an attempt to stabilize periodic orbits of the green laser, since the circuit could be easily operated in the microsecond time scale required for the laser.

The laser intensity was detected with a fast photodiode and this signal provided the input for the control circuit. The output of the control circuit modified the injection current of the diode laser used to pump the Nd:YAG crystal. This seemed to be the most natural and convenient choice of control parameter. To adapt Hunt's circuit for control of the autonomously chaotic laser, we had to supply an external timing signal from a function generator. This determined the interval T between "kicks" applied to the pump laser injection current. Even though there was no external periodic modulation responsible for the chaotic dynamics, the relaxation oscillations of the laser intensity provided a natural time scale for perturbative corrections. The interval between kicks was thus adjusted to be roughly at the relaxation oscillation period (approximately 100 kHz), or a fractional multiple of it. The OPF algorithm is shown schematically in Figure 2. The four parameters of the control circuit are: The period T, duration of the kicks dt, reference level I_{ref}, which measures the deviation of the signal, and the proportionality factor p, which determines the amplitude of the kicks.

The results of application of OPF to the laser were quite remarkable.[25] We were able to demonstrate stabilization of a large variety of periodic waveforms with perturbations of a few percent applied to the pump laser injection current. A typical chaotic waveform, together with several periodic waveforms stabilized in this way, are shown in Figure 3. The control signal fluctuations are shown above the intensity waveforms. The particular waveforms stabilized can be selected by changes of control circuit parameters, mainly the time period T and the reference level I_{ref}.

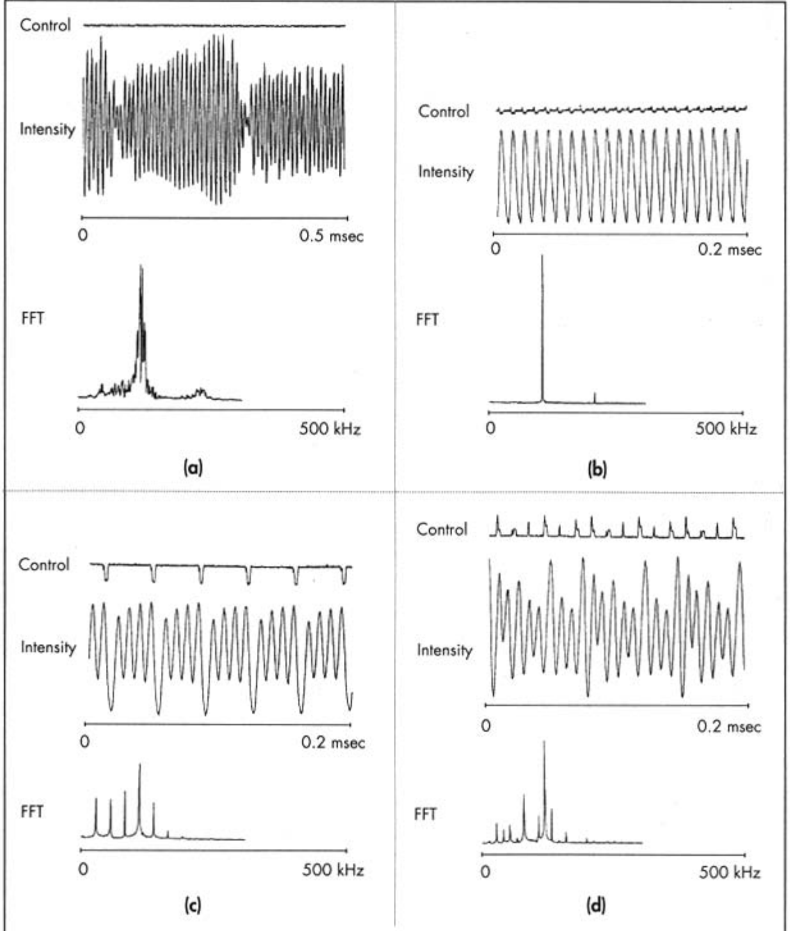


Figure 3

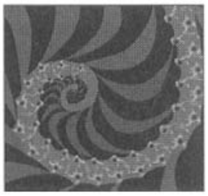
An example of stabilization of periodic orbits or waveforms of the chaotic laser. A typical chaotic time trace for the fundamental wavelength is shown in (a), in which the underlying relaxation oscillations are present. The control signals, waveforms and their fast Fourier transforms are shown in (b), (c), and (d) for three periodic orbits that have been stabilized. The relative fluctuations of the control signal about the ambient value are of the order of a few percent.[25]

For the control circuit to work successfully, the laser had to be operated so as to generate very little green light. The laser is "weakly" chaotic in this regime; the rate of separation of initially close trajectories in phase space is small, and only one direction of instability occurs. If a significant amount of green light was generated, and the laser was highly chaotic (particularly if the laser has more than one direction of instability in phase space), the circuit may be unable to stabilize the laser.

Stabilization of the Steady State

Of course, these experiments beg the question: If the laser is in a chaotic state, can we apply small corrective perturbations to obtain a stable output? This is, of course, interesting from an engineering standpoint and for practical application. Much to our own surprise, we found that we could indeed achieve a stable output by adjusting the reference level to the mean of the chaotic fluctuations and matching the period T to the relaxation oscillation period. The control voltage fluctuations became extremely small once the steady state was controlled. Figure 4 shows the transient behavior of the laser intensity fluctuations as they are reduced to small fluctuations about the steady state as well as the control signal fluctuations during the stabilization process.

If the control parameters are fixed and the pump power of the laser is increased or decreased after the steady state is stabilized, the control signal fluctuations increase rapidly, and control is lost as the laser goes into periodic or chaotic oscillations. Clearly, one needs to change the control circuit parameters as the laser pump power is changed. A procedure called "tracking" accomplishes this change of control parameters in a systematic fashion. We applied such a tracking procedure to our laser; the control circuit parameters are varied to minimize the control signal fluctuations at each value of the pump power, which is increased in small increments. Our experiment illustrated the general algorithms



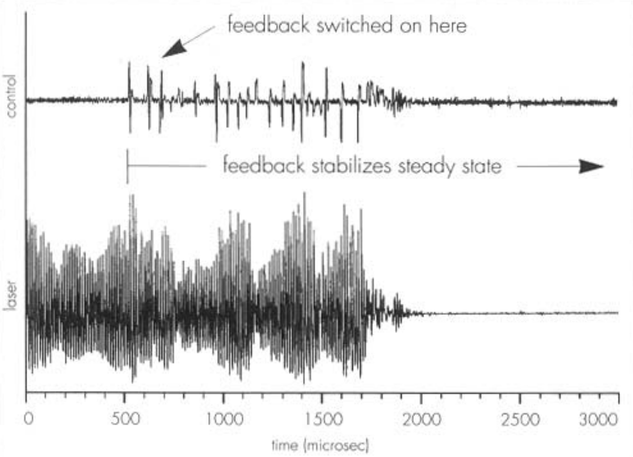


Figure 4

Stabilization of the steady state. The transients toward the steady state are shown when the control signal is applied to a chaotic laser. The chaos here is due to reflective feedback, not the intracavity crystal.

for tracking periodic orbits developed recently by Ira Schwartz and his colleagues at the Naval Research Laboratory.[26,27] By combining stabilization and tracking, we maintained a stable steady state (Fig. 5) as the laser pump power was increased from threshold (21 mW) to more than three times above threshold (about 80 mW).[28] Without the control circuit, the laser intensity went into periodic oscillations at a pump power of about 25 mW, and then into chaotic fluctuations, as indicated in the figure by open circles and crosses.

Current Research and Future Directions

Experiments on the control of chaotic lasers have been performed in several laboratories around the world. In a series of elegant experiments, Pierre Glorieux and his co-workers in Lille, France, have demonstrated both stabilization of periodic orbits and of the unstable steady state in an Nd:fiber laser.[29,30] Stabilization of the steady state was achieved by continuous derivative feedback in their experiments. They also demonstrated the tracking of unstable periodic orbits as system parameters were varied. Another experiment in Lille showed that the unstable branch of a bistable optical system could be stabilized by these feedback techniques.[31] The experiments of Brun and colleagues in Zurich on an NMR laser succeeded in systematically stabilizing several periodic orbits by an extension of the OGY technique.[32] Apart from these experiments, several groups have investigated the application of such control techniques to models of semiconductor laser diodes destabilized by optical feedback from an external reflector.[33,34]

The implementation of the control algorithm has been done electronically so far either by digital techniques or by analog hardware. An alternative approach, particularly if speed is crucial, may be the development of all-optical processing and feedback. To our knowledge, no experiments have demonstrated dynamical control and stabilization of chaos through purely optical techniques. Could neural networks (optically implemented, for speed) be used to predict future behavior of the chaotic system and help determine optimal corrections?[35]

In the OGY technique, the system must come sufficiently close to an unstable periodic orbit to stabilize it successfully with small perturbations. What if we don't want to wait for long periods of time, as is typically the case for complex waveforms? The Maryland group has developed a technique called "targeting," in which a chaotic system moves from its current state to a desired state in as short a time as possible through small perturbations. This is still a very active area of theoretical research, and the technique has yet to be implemented on an optical system or device. The targeting algorithm has been demonstrated experimentally on the magnetoelastic ribbon system by Shinbrot *et al.* with impressive results.[36]

The robustness of control techniques to external noise and drift of parameters is a crucial technical issue that will have to be investigated in detail for the practical implementa-

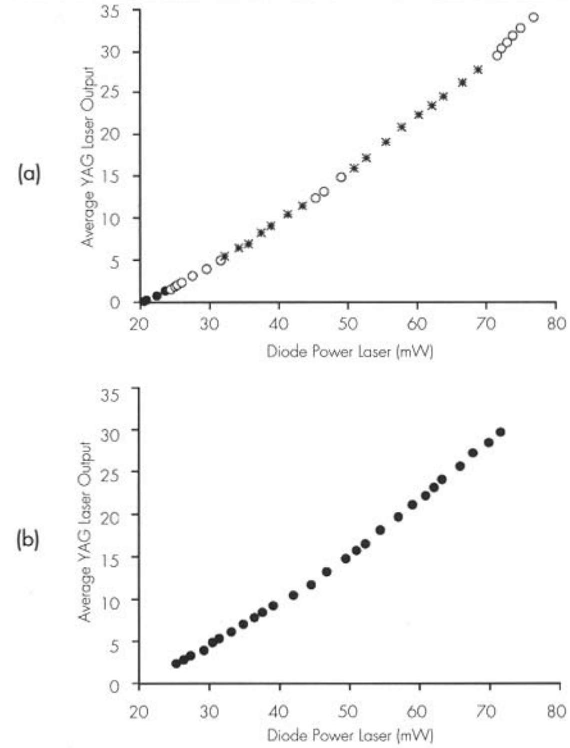


Figure 5

Stabilization of the steady state; control combined with a tracking algorithm allowed us to extend the stability regime of the laser. In (a) the solid circles represent stable output, the open circles periodic oscillations, and the crosses chaotic fluctuations; no control or tracking is done. In (b), stable output is obtained over the entire pump power range with control and tracking. The y-axis shows the Nd:YAG 1.06 μ output on a relative scale.[28]

tion of these techniques. On the more fundamental side, a difficult issue that is sure to emerge in the near future is the influence of intrinsic (quantum) noise on nonlinear dynamics and the resulting limitations on control of chaotic systems.

Finally, an important area of research in the future will be the control of chaotic systems with higher dimensional chaotic attractors (for example, those with more than one direction of instability in phase space) and sytems that possess both spatial and temporal degrees of freedom.[37,38] Laser arrays may be the test-bed for application for techniques that are being developed to control spatio-temporal chaos. Here the emphasis may be on the dynamical control of spatial profiles, including the periodic (or aperiodic) scanning of beams in space.

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