Royal Cabinet of Paintings Mauritshuis The Hague

Axicon—the Most Important Optical Element

Zbigniew Jaroszewicz, Anna Burvall and Ari T. Friberg

Johannes Vermeer is thought to have used a camera obscura for "*The Girl with a Pearl Earring*," c. 1665-1667, Mauritshuis, The Hague. The painting is widely regarded as one of Vermeer's masterworks.

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Last year, the optics community celebrated the 50^{th} anniversary of the formal naming of the axicon. Long before that, however, axicons generated vivid discussions and disagreements, often of fundamental importance to our understanding of optics. More recently, axicons have become key components in practical and scientific applications.

J. H. McLeod coined the word "axicon" in 1954 from the Greek words ἀζωνας (axis) and εικόνα (image).[1] According to his definition, an axicon is an optical element that images a point into a line segment along the optical axis. The axicon should be a figure of revolution and the object point placed on its symmetry axis (see Fig. 3, p. 37). The particular version proposed by McLeod was a glass cone, but considering his definition, almost any optical element with a rotational symmetry qualifies as an axicon. For example, a perfect lens is a special case of an axicon. Also by McLeod's definition, axicons were used and analyzed long before they given a formal name.

The title of this article should not be taken entirely seriously. Nonetheless, axicons—though unknown to most people—are important in optics. How can one measure their importance? Let us assume that the most important optical element is the one that has generated the most discussions, contradictions and disagreements. By that criterion, the axicon surely wins. The number of axicon applications has also increased significantly in the last few years—another indication of their importance.

Who discovered the pinhole camera and when?

Chronologically, the pinhole camera can be treated as the first axicon because, in principle, it possesses an infinite depth of focus. The question of who invented this oldest image-forming device still causes discussion among historians. The earliest mention of the pinhole camera seems to belong to the Chinese philosopher Mo-Ti (5^{th} century B.C.). Apparently, Aristotle (384-322 B.C.) knew the optical principle of the pinhole camera's action. He observed a partially eclipsed sun through the holes in a sieve and questioned how the sun could make a circular image when it shines through a square hole.

Others give priority to the Arabian scientist Alhazen (ca. 965-1040 A.D.), considered one of the great optics authorities of the Middle Ages. He described applying the pinhole camera to safely observe solar eclipses without endangering the eyes, and he used it as an analog to show how an image is formed in the eye. Roger Bacon, who also adopted the pinhole camera for the eclipse of 1247, brought Alhazen's work into the European tradition.[2]

During the European renaissance, the pinhole camera became an important device both in science and art (Fig. 1). In 1490, Leonardo Da Vinci described it in his notebooks and may have used it later for his studies of perspective. Apparently, Giambattista della Porta (1535-1615) was the first European to publish information about the pinhole camera, and therefore, he is sometimes incorrectly credited with its invention. He may have coined the term "camera obscura," which in Latin stands for dark room, but della Porta probably based his work on earlier books.[2] However, artists immediately took it up as a drafting tool, and there is a long and more than secular discussion about whether Vermeer used a camera obscura as an aid to render perspective (see facing page).[3] In the 17th and 18th centuries, the pinhole camera became a standard device for painters.

To add to the confusion of the camera's origin, the nomadic tribes of North Africa, who lived in animal skin tents, probably knew of the image-forming ability of a tiny hole thousands of years ago. A pinhole in a tent would project an image of the scene outside.[2]

Poisson or Arago spot?

Perhaps the most important axicon controversy was the famous affair of the Poisson or Arago spot—a bright spot created by interference in the center of the shadow of an opaque disc or sphere.

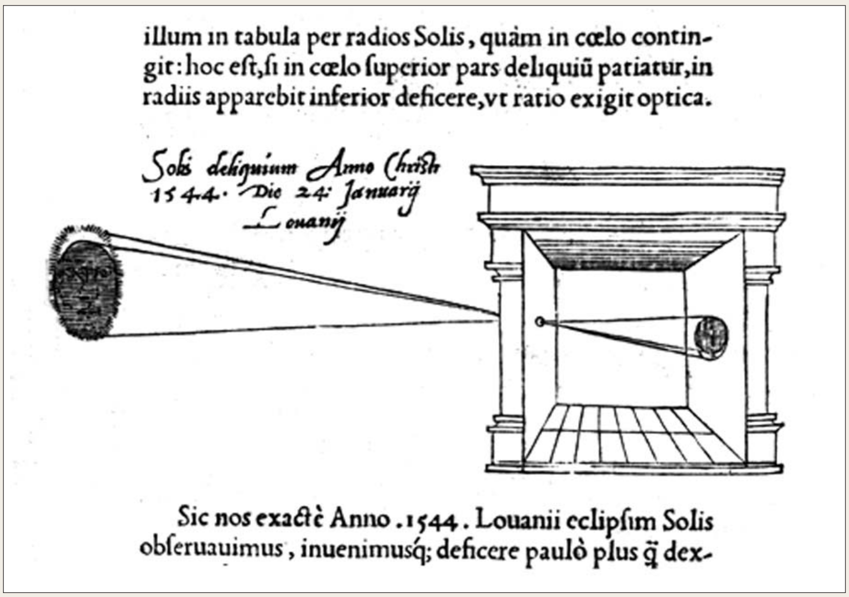


Figure 1. Camera Obscura, Reinerus Gemma-Frisius, 1544. Note that the figure is dated earlier than della Porta's work. [See www.acmi.net.au/AIC/CAMERA_OBSCURA.html.]

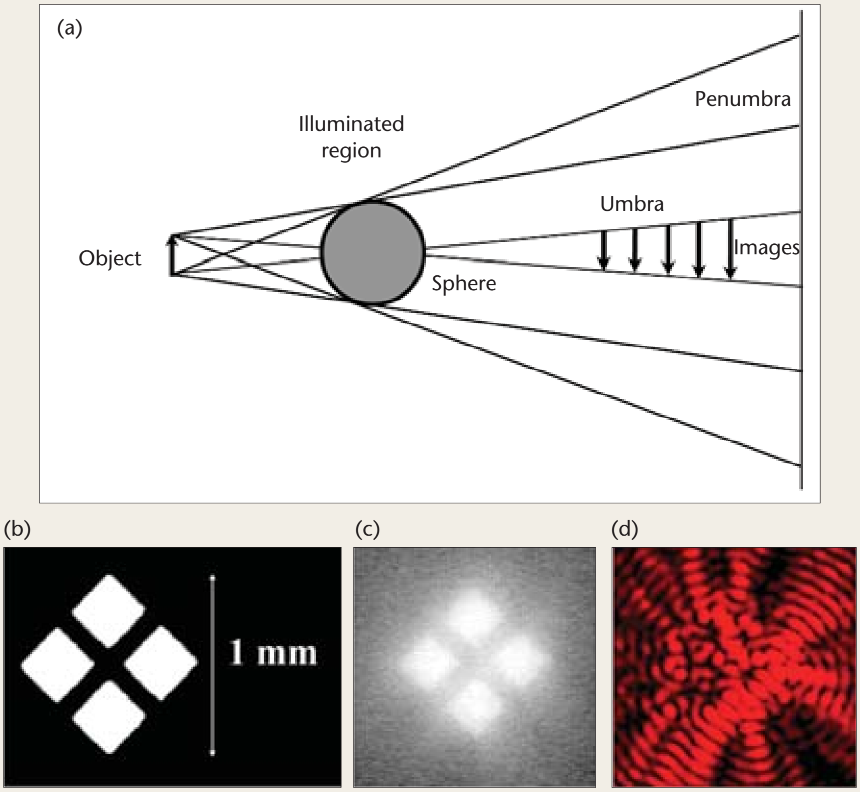


Figure 2. Image formation in the full shadow zone of an opaque sphere: (a) geometry of the experiment, (b) object, (c) image formed in white light and (d) image using a laser beam. [Ref. 4]

Even its name is not definitely established. It is sometimes referred to as the Arago spot (a term for which Google found about 8,000 Web pages at the end of 2004), and sometimes as the Poisson spot (about 66,000 Web pages). The Arago spot fulfills the definition of an axicon; it is possible to use an opaque spherical obstacle as an image forming device (Fig. 2).[4]

It is well known that Poisson, an adherent of the corpuscular theory of light, noticed that a consequence of the wave theory presented by Fresnel to the contest of the French Academy in 1818 should be such a bright spot. Because this was counter-intuitive, he discarded the whole wave theory as incorrect. When Arago experimentally confirmed this result, which at first sight seems contrary to common sense, Fresnel's theory gained strong experimental support and the corpuscular theory was refuted. From that time on, the episode has served philosophers of science as a favorite example of "experimentum crucis," a crucial experiment that puts a theory to the test.

Nevertheless, it seems that this fundamental controversy has been modified over the years to fit the views and needs of those who tell it. It is worth mentioning, however, that Fresnel obtained the French Academy prize for his work before the experimental results of Arago became widely known (or at least published), and that the contest's commission assigned the award to Fresnel despite the lack of experimental support for his theory. Interested readers should consult the excellent monographs of J. Z. Buchwald and Nahum Kipnis for more information.[5]

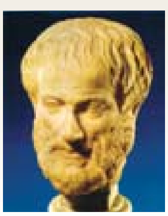
Predating this famous incident, G. F. Maraldi probably observed the Arago spot for the first time almost a century earlier.[6]

Diffraction-free beams or subject to diffraction?

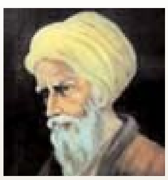
The Bessel beam, first proposed 18 years ago, provides a more recent example of axicon imaging.[7] Following the beam's first presentation, a hectic discussion regarding its nondiffracting qualities took place.[8] Usually a nondiffracting beam is

MILESTONES IN AXICON HISTORY

5th century B.C.
Earliest mention of the pinhole camera made by Chinese philosopher Mo-Ti.

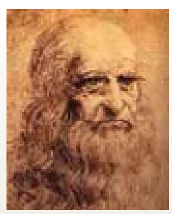


384-322 B.C.
Aristotle observes a partially eclipsed sun through holes in a sieve and questions how the sun could make a circular image when it shines through a square hole.



965-1040 A.D.
Arabian scientist Alhazen describes how to use the pinhole camera to safely observe solar eclipses.

1247 A.D.
Roger Bacon adopts the pinhole camera for the eclipse of 1247, bringing Alhazen's work into the European tradition.



1490 A.D.
Leonardo Da Vinci describes the pinhole camera in his notebooks.

DEVELOPMENT OF THE PINHOLE CAMERA →

AXICONS IN AR

understood as a solution of the wave equation separable into two parts; one depends only on the propagation coordinate, whereas the other is a function of the transverse coordinates and remains unaltered for any position along the optical axis. Obviously, in the case of an infinite aperture, the Bessel beam, described as $J_0(k\alpha\rho)\exp(ik\gamma z)$—where $k=2\pi/\lambda$, λ is the wavelength, and $\alpha^2+\gamma^2=1$ (α and γ are the radial and axial direction cosines)—fulfills the definition of a diffraction-free beam.

Nevertheless, it was claimed that even with a finite aperture, the Bessel beam maintains much of its nondiffracting character.[8] Supporters of this view based their argument on the comparison of a Bessel beam's invariance range with a Gaussian beam of waist equal to the Bessel beam's central lobe. Of course, the Rayleigh range of the Gaussian beam is much shorter than the length of the Bessel beam's focal segment, but one can hardly view this as proof of the Bessel beam's increased resistance against diffraction. In a Bessel beam, the energy flow is not axial, but takes place under the angle θ (Fig. 3).

Therefore, successive points along the focal segment are powered by annuli of increasing radius on its aperture, outside the width of its central lobe. In other words, the Bessel beam is an interference pattern, in which the outer rings continually maintain the central maximum and prevent it from spreading (Fig. 3). When isolated, the central lobe of the Bessel beam would spread out as quickly as a Gaussian beam of the same waist.

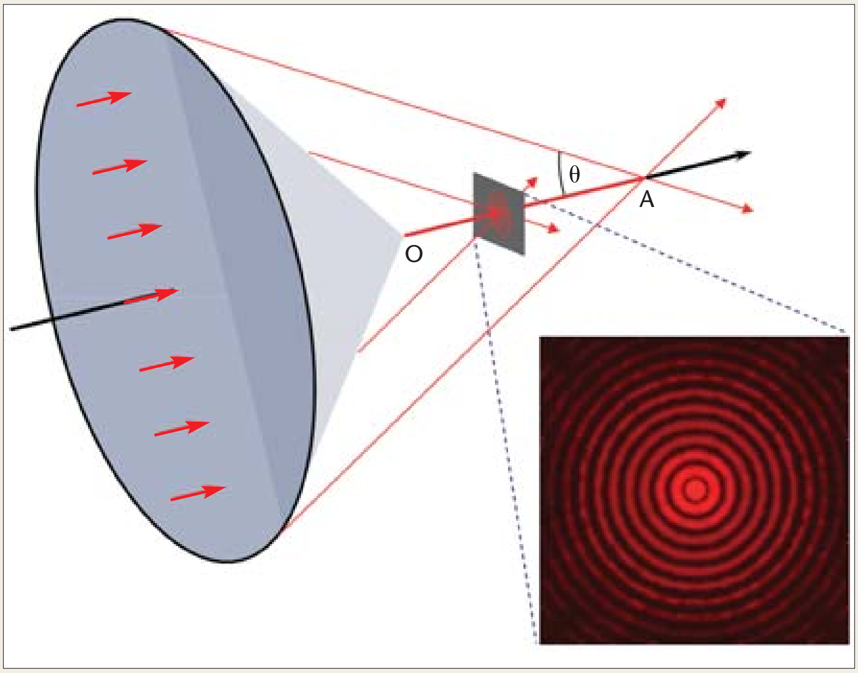


Figure 3. Focal segment OA formed by a classical linear axicon described by J. H. McLeod [Ref. 1] and the cross-section of the focal segment. The radius of the central peak is given by $\rho_0=c\lambda/\alpha$, where $\alpha=\sin\theta$ and $c=0.3827$. [Ref. 12]

Superluminal velocity or not?

The most recent axicon controversy regards the superluminal velocity of the Bessel beam. A short session on the Internet reveals a huge number of papers dealing with this subject. We mention just a few of them in order to give some examples.

One experiment in which a superluminal velocity was announced is interesting because of its historical context. It deals with the axial velocity of the interference fringes created by a boundary wave from an opaque sphere (an Arago spot), superposed with a spherical wave originating from the center of the drilled sphere. Nevertheless, this is an example of phase velocity, which can be superluminal. In another experiment, an ionization front produced

1600s
The pinhole camera becomes a standard device for painters.

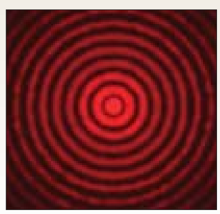
1818
Poisson notices that a consequence of Fresnel's wave theory should be the bright spot that comes to be named for him and Arago, who experimentally confirms this result.

1954
J.H. McLeod introduces the name "axicon."

WAVE-OPTICAL AXICON IMAGES — MODERN AXICONS

1535-1615 A.D.
Giambattista della Porta is the first European to publish information about the pinhole camera. He may have coined the term camera obscura.

1700s
G. F. Maraldi probably observes the Poisson or Arago spot—a bright spot created by interference in the center of the shadow of an opaque disc or sphere—almost a century before Poisson and Arago.



1980s
The diffraction free Bessel beam provides a recent example of axicon imaging.

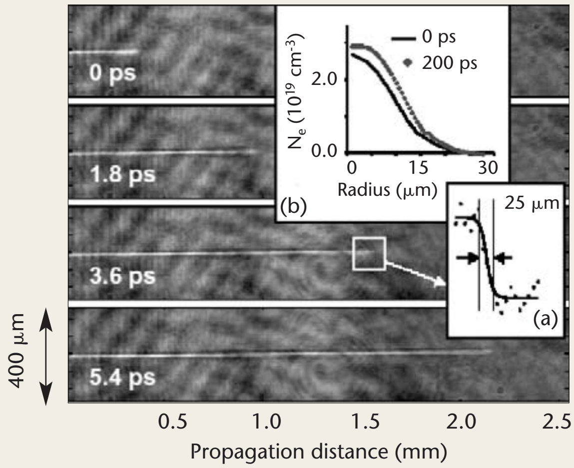


Figure 4. Ionization tracks produced by an advancing ultrashort (~70 fs) optical Bessel beam pulse at four successive times. The propagation range of the pulse is about 9.2 mm. The ionization front moves at the Bessel pulse's superluminal group velocity. Insets: (a) axial profile of the ionization front; (b) transverse electron density profiles at two time delays. [Ref. 9]

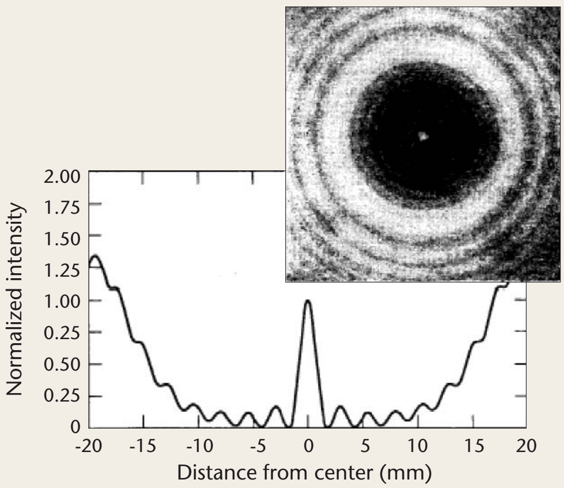


Figure 5. Poisson line for FEL alignment. The inset shows the Poisson (or Arago) spot from a 25 mm diameter sphere illuminated by a laser, in observation plane at the distance of 100 m. The curve is the calculated intensity profile out to the edge of the shadow. [Ref. 13]

by the focal segment of a pulsed Bessel beam turned on in an ambient gas was measured to be superluminal (Fig. 4).[9]

One proposed interpretation refers to the so-called scissors effect. Although the velocity of the scissor blades is subluminal, their contact point can move faster than light. This analogy is immediate if one takes into account that the Bessel beam can be considered as an azimuthal superposition of plane waves of equal inclination with respect to the optical axis. In other words, its spatial-frequency spectrum is a circle.

However, there is no possibility of obtaining superluminal communication because every point of the focal segment is independent of its neighbor, created by the corresponding aperture annulus. To establish communication between two points on the image segment, the signal must first move between the respective annuli in the aperture and then proceed to the optical axis. When this is taken into account, the communication velocity turns out to be subluminal and the causality law saved. The whole issue of superluminal velocity is actively debated in the literature and has passionate opponents[10] and proponents.[11]

Modern applications

The Bessel beam neither breaks the laws of diffraction nor enables superluminal communication. Nonetheless, the classic axicon line image (as the Bessel beam used to be called) has a number of interesting applications. The most important uses of Bessel beams were, and still remain, in alignment activities. Axicons form a long and narrow light segment along the optical axis and serve first of all as elements defining a reference line. This application is fully justified when one takes into account that the ratio of the diameter of the central peak to the length of the focal segment is comparable to λ/R, where R is the aperture of the element.[12] This allows for accuracy on the order of 10^{-6}.

The list of metrological applications covers possibilities such as the alignment of optical systems and mechanical devices, testing the flatness of large surfaces, detecting small objects by observing scattered light, optical triangulation and sectioning. We illustrate here one particular example, which is interesting because of its historical contexts; namely, the oldest wave-optical axicon image, the Arago spot, is proposed for the alignment of a modern free electron laser (Fig. 5).[13]

Another important application of axicons lies in imaging with an increased depth of focus. Logarithmic axicons are indicated as the best ones for this particular task, because the optical intensity remains constant along the focal segment.[14]

Optical tweezers for the manipulation of micro-objects were proposed simultaneously with the Bessel beams, and soon the use of axicon lines for particle trapping was observed and appreciated.[15] In recent years, optical tweezers have attracted much attention, and they have been converted into standard devices for the positioning of biological cells.

Investigators have obtained more-complex axicons producing higher-order Bessel beams, twisted beams and optical vortices by adding an angular phase term to the transmittance.

The focal segments of such wave fields show, apart from interesting topological properties, a pipeline intensity and a rotating phase structure that can exert torque on trapped particles and rotate them as a result of angular momentum. Optical spanners—primarily making use of orbital rather than intrinsic or spin angular momentum, and of Laguerre-Gaussian beams instead of fields attributed to the optical twist phenomenon[16]—were obtained in this way (Fig. 6).

Another rapidly developing application area in which axicons play a key role is nonlinear optics, particularly optical frequency doubling by Bessel beams, a use initiated by the seminal work of Wulle and Herminghaus.[17] Related topics include the pumping of active media and dispersion compensation. A recent proposal suggests the use of an axicon in optical coherence tomography for recovering the three-dimensional structure of the human eye.[18]

The types of axicons available—and the variety of the fields they generate—have grown considerably in recent years. Apart from the Bessel beams, new continuous wave and pulsed solutions have emerged, such as the X-waves, Matthieu beams and parabolic nondiffracting fields. Some of them allow generalizing the axicon definition and ignoring the requirement of radial symmetry.[19] Fabrication technology has also advanced. Besides the classical cone axicons and the conventional Bessel beam set-ups, a variety of diffractive counterparts consisting of ordinary lenses and other variants flourish.

Hopefully, we have demonstrated that axicons are among the oldest known optical elements. Despite their long and turbulent past—or maybe thanks to it—axicons still offer novel, exciting possibilities in research and applications. Most important, we are convinced that as axicons embark on the second half-century of their formal existence, they will give rise to useful discussions of fundamental importance in optics.

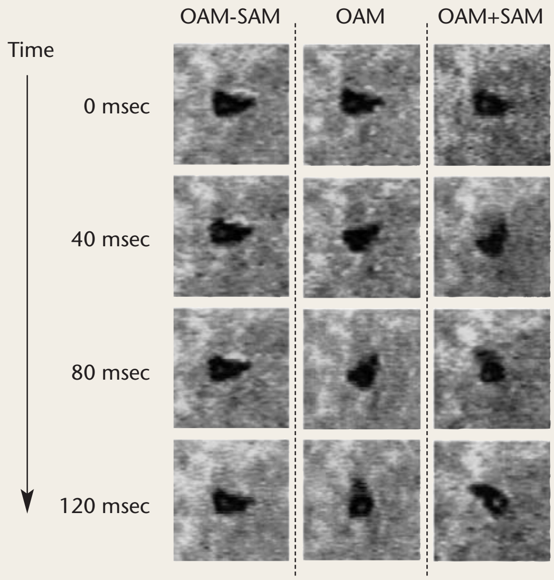


Figure 6. A light beam with a rotating polarization vector carries spin (SAM), whereas a beam that rotates as a whole carries orbital angular momentum (OAM). Both spin (1ħ per photon) and OAM (*n*ħ per photon) can on their own set a microparticle into rotation (center and right columns). If the beam's OAM is 1ħ per photon and if the beam is oppositely circularly polarized (spin = -1ħ per photon), the rotation stops (*left column*). [See www. physics. gla.ac.uk/Optics/projects/opticalSpanner/]

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